

# Zone based Optimal Reactive Power Dispatch in Smart Distribution Network using Distributed Generation

Aadil Latif, Ishtiaq Ahmad  
Center for Energy  
Austrian Institute of Technology (AIT)  
Vienna, Austria  
{givenname.lastname}@ait.ac.at

Peter Palensky  
Faculty of Electrical Engineering,  
Mathematics and Computer Science  
TU Delft, Netherland

Wolfgang Gawlik  
Institute of Energy Systems  
and Electrical Drives  
TU Wien, Austria

**Abstract**—Increased penetration of distributed generation in power distribution network influences voltage profile in power distribution network. As voltage is a local phenomenon, active power injection from DERs only effects voltage of buses in close proximity. The voltage regulation problem can be solved effectively by dividing network into voltage control areas with high voltage interdependency. This paper proposes zones for voltage control through optimal reactive power dispatch for distributed generation to regulate voltage in the network. Zones were identified by using hierarchical agglomerative clustering algorithm based on voltage sensitivities to reactive power. A meta-heuristic algorithm was used for optimally determining the reactive power dispatch set points for distributed generators in each zone. Modified IEEE 37 node MV test network with distributed generation was used to evaluate the proposed voltage control scheme by using a co-simulation platform. Results showed that zone based reactive power dispatch is effective for improving voltage profile in the network.

**Index Terms**—Optimal reactive power dispatch, Hierarchical clustering, Voltage control zone, Distributed generation

## NOMENCLATURE

$g_k$	Conductance of the $k^{\text{th}}$ line.
$U_k$	Actual voltage at the $k^{\text{th}}$ bus.
$u_k$	Per unit voltage at the $k^{\text{th}}$ bus.
$u_{max}$	Maximum per unit voltage in the network.
$u_{min}$	Minimum per unit voltage in the network.
$\delta_{kl}$	Voltage angle difference for $k^{\text{th}}$ and $l^{\text{th}}$ bus.
$P_{loss}$	Total active power losses within the network.
$Q_{gk}$	Reactive power set point for the $k^{\text{th}}$ generator.
$\cos(\theta)$	Symbol denotes power factor of the generators.
$S$	Equipment loading as percentage of nameplate rating.
$u^{\cdot, l}$	Super scripts denote upper and lower limits for a parameter.
$u_{uv,lv}$	Upper and lower voltage bound violation index.
$\zeta$	Amplification constant.
$J_4$	Reactive power to voltage sensitivity matrix.
$\alpha$	Attenuation matrix.
$D$	Electrical distance matrix.
$D^{norm}$	Normalized electrical distance matrix.
$C_k$	Represents the $k^{\text{th}}$ cluster.

## I. INTRODUCTION

Power distribution networks are experiencing operational and management challenges due to increased penetration of distributed generation. This growth in use of distributed generation is due to many factors. These include but are not limited to; increased use of renewable generation, environmental consideration, technological developments. Stochastic active power injection from DERs can result in voltage variation. For example, a sudden increase in injection of active power may raise the voltage in the network which may result in violation of the upper allowable voltage limit. Traditionally on load tap changers (OLTCs) on HV/MV transformers have been used to regulate the voltage in the distribution network. The introduction of distributed generation however has reduced the voltage control capability of an OLTC. Reactive power compensation from distributed generators is an alternate voltage regulation method. Voltage problem is local in nature and should ideally be catered to using local resources. By dividing the network in independent voltage control zones a local voltage problem can be solved locally. Instead of having one central controller for the whole network, each zone can have separate controllers and can work independently. This helps in reducing the problems associated with centralized control schemes i.e. single point failure, increased communication and information flow to the control center.

Studies on optimal reactive power dispatch (ORPD) for voltage regulation is carried out by many researchers. Some of them are in [1], [2], [3], [4], [5] and [6], however, these studies focused only on centralized reactive power dispatch. Voltage control area concept is also investigated in some studies. The authors in [7] proposed voltage control areas for reactive power dispatch as an ancillary service in market. Voltage control areas on the basis of electrical distance were defined. The proposed market oriented design claims to improve the fairness of market with economic point of view. In another study [8] the authors have proposed adaptive zone based voltage control. Control zones were not fixed and were reconfigured on any structural variation in the grid. Improvements with the

proposed methodology was demonstrated by the field tests. A similar study was carried out in [9], and automatic voltage control algorithm was proposed with adaptive zone formation. The authors in [10] proposed voltage control (VCA) areas for reactive power dispatch for technical as well as economical issues. Objective function for each VCA was formulated to manage the reactive power reserve and the proposed scheme was effective for local var requirement. The authors in [11] presented a similar concept of Mvar control space based on sensitivity of voltage with respect to change in reactive power. A clustering algorithm was used for partitioning the power network.

## II. PROBLEM FORMULATION

In power systems, total active power transmission losses are calculated by summing losses for all branch elements. Active power loss in a branch element is a function of its conductance  $g$ , the voltage at its either end  $U_1, U_2$  and difference between voltage angles  $\delta_{12}$ . Total active power loss within a network can therefore be calculated using Eqn (1).

$$P_{loss} = \sum g_k (U_k^2 + U_l^2 - 2U_k U_l \cos \delta_{kl}) , \quad (1)$$

Active power loss is a function of node voltage, which in turn is a function of active and reactive power. In HV and MV networks X/R ratio is typically high and active power has limited influence on node voltage. Reactive power compensation is common practice for voltage regulation in MV distribution networks. Loss minimization may require inverters installed with the distribution network to operate in leading power factor and supply reactive power locally. This result in voltage violations as voltage limits may exceed bounds prescribed by international standards such as EN 50160 [12]. Ensuring voltage bounds are not violated is the responsibility of the distribution system operator (DSO). In this work, the voltage regulation objective has been made part of the overall objective function. The voltage violation indices  $u^{uv}$  and  $u^{lv}$  have been calculated using the following equations;

$$u^{uv} = \begin{cases} u^{max} - u^u & \text{if } u^{max} > u^u \\ 0 & \text{else} \end{cases} , \quad (2)$$

$$u^{lv} = \begin{cases} u^l - u^{min} & \text{if } u^{min} < u^l \\ 0 & \text{else} \end{cases} , \quad (3)$$

The final objective function has been constructed by summing the two individual objectives namely the loss minimization objective and the voltage regulation objective. The optimization problem is subject to constraints pertaining to inverters capability and operation limits detailed in international standards like VDE-AR-N 4105 [13] and IEEE STD 1547 [14]. It should also satisfy the constraints pertaining to power balance and safe system operation e.g. transformer overloading. The German grid integration standard requires DERs to support power factors of 0.9 in MV and LV networks. This power factor requirement visually shown in Figure 1.

Eqn. 4 presents the mathematical expressions for the proposed method.

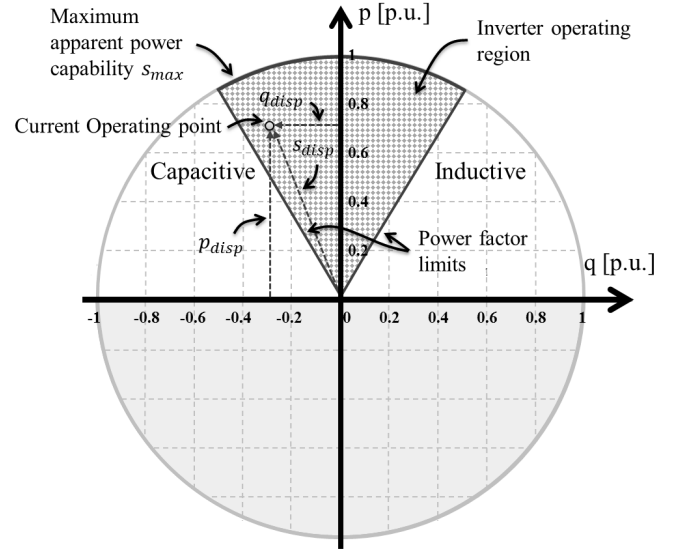


Fig. 1. Scree plot for the dendrogram

### • Objective Function:

$$\text{Minimize } F(\vec{Q}_{gi}) = P_{loss} + e^{\zeta(u^{uv} + u^{lv})} \quad (4)$$

where  $Q$  is the reactive power for distributed generator  $i = 1 \dots n$

### • Constraints:

– **Power balance constraints:** Pertain to law of conservation of energy

$$\sum S_{G_k} = \sum S_{d_k} + S_{loss}$$

– **Transformer thermal constraints:** Thermal limits of the distribution transformer

$$S_{tr} \leq S_{tr}^u$$

– **Generation Reactive Power Constraints:** Generator reactive power output constraints (lower and upper limits of  $Q$ )

$$Q_{g_k}^l \leq Q_{g_k} \leq Q_{g_k}^u$$

where  $i = 1, 2, 3, \dots$ , total number of generators

– **Generator Power factor constraints:** Upper and lower power factor limits for a generator

$$\cos(\theta_{g_k}^l) \leq \cos(\theta_{g_k}) \leq \cos(\theta_{g_k}^u)$$

where  $i = 1, 2, 3, \dots$ , total number of generators

## III. ZONE FORMATION

This work is an extension of the work presented in [6]. In the previous work results for centralized implementation were presented and discussed in detail. It was shown that the proposed method for optimal reactive power dispatch resulted in improved system efficiency. Centralized approach can be a bottleneck in real world implementation as it has a number of limitations. These include but are not limited to the following

- **Prone to single point failure** - Centralized control approach is susceptible to single point failure, that is, if the central coordination controller fails the entire system fails. Traditionally, redundant communication channels, sensors and other auxiliary equipment have been used by TSOs at the HV level. Due to the sheer number of MV distribution grids, this practice may be infeasible for MV networks. Decentralized implementation would ensure that even if one of the controller fails, the entire network is not affected.
- **Does not supports deregulation** - A number of countries around the world are moving towards deregulated energy markets. Any new method should therefore support deregulation. Unlike the centralized approach, decentralized implementation supports deregulation as each as zone has an independent autonomous controller.
- **Scalability issues** - Another main bottle neck in centralized implementation is scalability issue. In a centralized implementation, the coordinating controller communicates with all the controllable resources. By defining zones each with an independent voltage controller, The proposed method can overcome the scalability issue.
- **Requires less information exchange** - In decentralized implementation each controller communicates with resources within the zone. This results in reduction in information exchange overhead. Hence, cheaper communication technologies can possibly be used, thus reducing implementation costs.

Although there is a definite need to move from centralized implementation towards decentralized implementation, there are a couple of drawbacks that warrant a mention.

- **Conflicting objectives** - As every zone controller runs autonomously and independently, it has no knowledge of neighboring zones. It is a possibility that at a point in time neighboring zones might be optimizing conflicting objectives. This can potentially result in a degradation in the performance of the coordinating zone controllers.
- **Convergence to local optimum** - Finally, each zone controller converges to a local optimum as it only has zones information. The global optima might differ for the local optima. A large difference between the two optima will result in reduced performance.

#### A. Methods for zone formation

There is no unique way for identification of zones, however, electrical distance can be used for zone formation. Electrical distance can be calculated by the bus admittance matrix or sensitivity matrix. In [15] the authors have proposed a K-means clustering based approach to define voltage control regions for efficient reactive power management. Similarly in [16], the authors have proposed two clustering based zoning algorithms that have been tested on the IEEE 13 node test feeder and the IEEE 123 node test feeder. In this work clustering based zoning has been used to define zones within the chosen network.

#### B. Calculation of normalized electrical distances

A power system can be represented by a set of linear equations known as the Jacobean matrix. Popular load flow method solvers like Gauss-Seidel and Newton-Raphson use the Jacobean matrix iteratively to converge to a solution. The Jacobean matrix is constructed using four sub matrices. The fourth sub matrix commonly referred to as  $\partial Q/\partial u$  or  $J_4$  is real and unsymmetrical. The inverse of  $J_4$  is used to calculate the sensitivity matrix, which is also real and asymmetrical. In [7] the authors have proposed method for calculating the normalized electrical distance within a network. The inverse of  $J_4$  is used to calculate the voltage/reactive power sensitivity matrix  $B$ . The sensitivity matrix  $B$  quantifies the impact of varying reactive power in the network on any node's voltage. Next, attenuation matrix is calculated by dividing the non diagonal elements by the diagonal elements. In the next step relative electrical distance is calculated. In the final step the distance matrix is normalized. The steps are as follows;

- 1) Calculate the Jacobian matrix and use it to obtain the  $\partial Q/\partial u$  matrix.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial u} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial u} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta u \end{bmatrix}, J_4 = \frac{\partial Q}{\partial u} \quad (5)$$

$$B = J_4^{-1} = \frac{\partial u}{\partial Q} \quad (6)$$

- 2) Calculate the sensitivity matrix  $B$  by calculating the inverse of  $J_4$ .

$$B = J_4^{-1} = \frac{\partial u}{\partial Q} \quad (7)$$

where,

$$b_{ij} = \frac{\partial u_i}{\partial Q_j} \quad (8)$$

- 3) Calculate the attenuation matrix by dividing the non-diagonal elements by the diagonal elements using the following equation;

$$\alpha_{ij} = \frac{b_{ij}}{b_{jj}} \quad (9)$$

- 4) Calculate the relative electrical distance using Eqn. (10) and obtain the normalized distance matrix Eqn. (11)

$$D_{ij} = -\log(\alpha_{ij} \cdot \alpha_{ji}) \quad (10)$$

$$D_{ij}^{norm} = \frac{D_{ij}}{\max(D_i)} \quad (11)$$

Fig. 2 is colour plot for the normalized electrical distance (NED) for the IEEE 37 node test feeder. After the NED has been obtained either visual inference or a clustering method can be used to define zone boundaries within the network. Visual inference can be error prone specially for larger networks. In this work, hierarchical clustering has been used for defining zone boundaries.

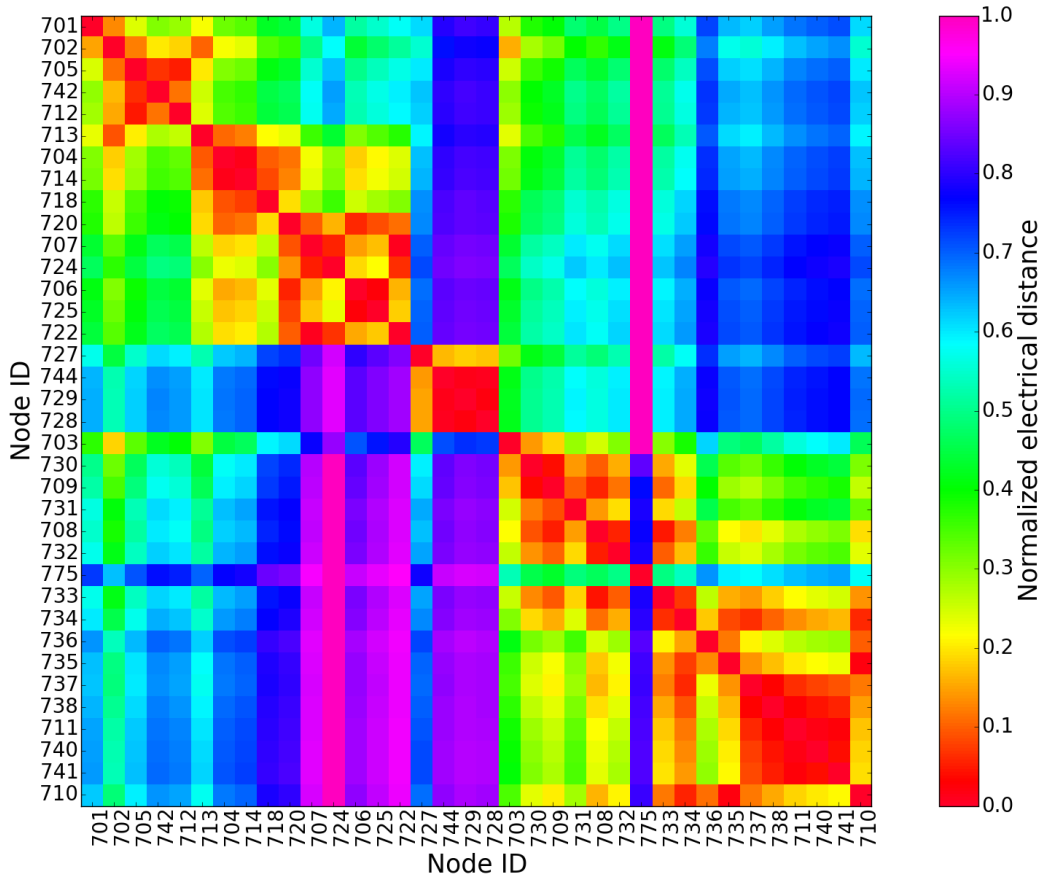


Fig. 2. Color plot of the normalized electrical distance matrix for the IEEE 37 node test feeder

### C. Hierarchical agglomerative clustering

Hierarchical agglomerative is an iterative clustering algorithm that creates a hierarchy of clusters following a bottom-up approach. Initially  $n$  clusters are created, (where  $n$  is the number of elements that need to be clustered) each containing only one element. Clusters are then iteratively combined until only one cluster remains containing all elements. In order to decide which two clusters should combine each iteration, a measure of dissimilarity is used. Of the remaining clusters, two clusters the least dissimilar to one another are combined at each step. The dissimilarity measure can be calculated by choosing a linkage method most suitable for the problem. In this work ward linkage has been used.

Ward minimum variance method aims at minimizing variance within clusters. The variance is calculated by adding the sum of squares over all variables belonging to the two clusters. The two clusters with least variance are selected each iteration. The distance between the two clusters is defined by,

$$d(C_x, C_y) = \|C_x, C_y\|^2 \quad (12)$$

Where  $C_x, C_y$  are two predefined clusters. Algorithm 1 presents the pseudo code for hierarchical agglomerative clustering with ward linkage.

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**Algorithm 1** Pseudo code for hierarchical agglomerative clustering with ward linkage

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**Input :** An  $n \times n$  distance matrix  $D$

**Result:** Dendogram and Scree plot

**while** Number of clusters  $\neq 1$  **do**

Find the least dissimilar pair of cluster  $C_a, C_b$  where  $a, b \in [1, \dots, n]$  according to ward linkage criterion

Increment the sequence number  $=+1$ . Merge clusters  $C_a, C_b$  into a single cluster. Set the level of clustering  $L(k) = d(C_a, C_b)$

Delete the rows and columns corresponding to  $C_a, C_b$

Add row and column corresponding to newly formed cluster  $C_a, C_b$ , calculated using the equation  $d[C_o, C_a + b] = \min(d[C_o, C_a], d[C_o, C_b])$

**end**

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From the dendogram in Fig. 3, at dissimilarity equal to 2, six tight clusters are clearly visible. The knee point in the Scree plot also occurs at  $k = 6$ , which makes six clusters a good choice for the number of clusters. Node 775 however has been identified as a cluster containing a single element. One

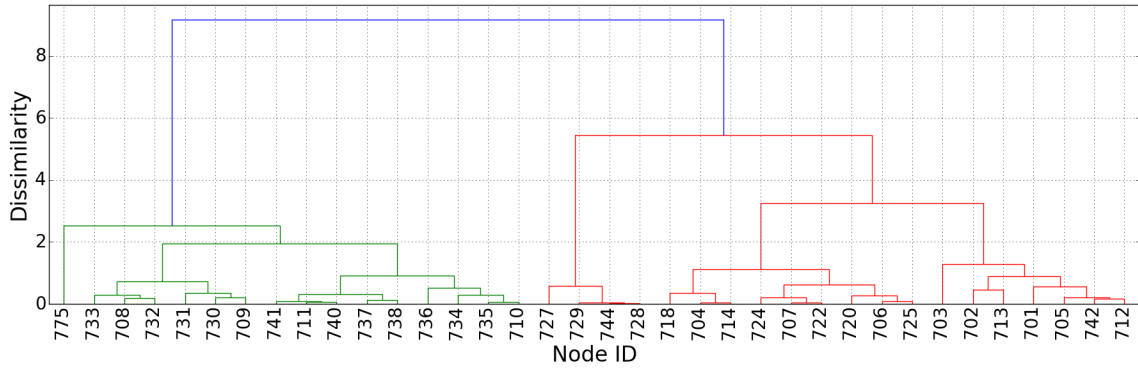


Fig. 3. Dendrogram for clusters produced using ward linkage based agglomerative hierarchical clustering

criterion for an acceptable zone is that it should contain at least one node connected to a DER; otherwise, parts of the network would be unregulated due to lack of controllable devices. For this reason in this work node 775 has been grouped with the neighboring zone. Table I shows the nodes clustered in each zone.

#### IV. CASE STUDY

The test case chosen for conducting the simulations for the proposed method is the IEEE 37 node MV test network [17]. The network configurations as presented in [6] have been used for this work.

##### A. Co-Simulation setup for study case

Co-simulation is an effective way for simulation of a system involving multi-domains. DigSILENT PowerFactory was used as a power system simulator which allows various methods for coupling with other tools. Details are available in [18], [19], [20]. The proposed method has been implemented by coupling Python, a powerful open source scripting language with a large number of multi domain libraries, and DIgSILENT PowerFactory. The IEEE 37 node test feeder has been implemented in PowerFactory, while the meta-heuristic optimization algorithm has been implemented in Python. PowerFactory is additionally used as a load flow engine to calculate the fitness value of a solution generated by meta-heuristic optimization algorithm.

#### V. RESULTS & DISCUSSION

In this work, five reactive power based voltage regulation schemes have been implemented on the IEEE 37 node test feeder. Comparative results show that fixed power factor control has a significant impact on peak voltage seen in the network causing the peak voltage to drop by 0.039 p.u. (Figure 5). Similarly, variable power factor results in a drop of 0.035 p.u. in the peak voltage appearing within the network. Volt var control (VVC) is a much more efficient method of local voltage control. It significantly reduces unnecessary RPC thereby reducing losses and improving system efficiency. Peak voltage experienced by centralized and decentralized implementations of coordinated control schemes is very similar. Figure 6 is the plot of the energy saved compared to base

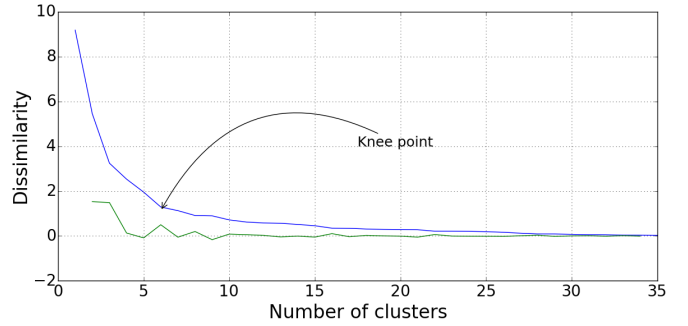


Fig. 4. Scree plot for the dendrogram

TABLE I  
LIST OF NODES GROUPED IN A CLUSTER USING HIERARCHICAL CLUSTERING

Zone 0	Zone 1	Zone 2	Zone 3	Zone 4
708	710	727	704	701
709	711	728	706	702
730	734	729	707	703
731	735	744	714	705
732	736		718	712
733	738		720	713
775	738		722	742
	740		724	
	741		725	

case implementation (no voltage control). As can be seen from Figure 6, both fixed power factor and variable power factor control result in a significant increase in active power losses. VVC scheme is a more efficient RPC scheme that in this case results in considerable reduction in losses. It should be noted that for both centralized and decentralized implementations of coordinated control net energy saved is positive and equal. With intelligent zoning and decentralized implementation, it is possible to achieve results comparable to centralized implementation for the proposed optimization method.

Both centralized and zone based implementation of the

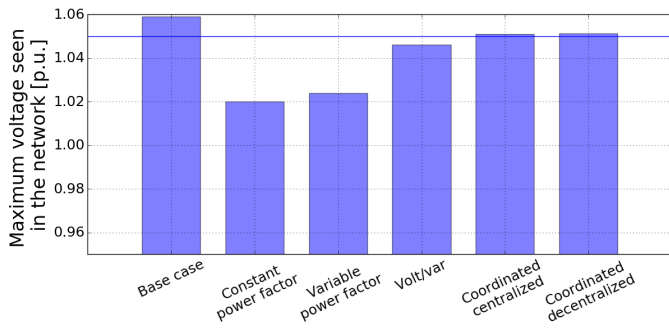


Fig. 5. Comparison of maximum voltage seen in the network by implement.

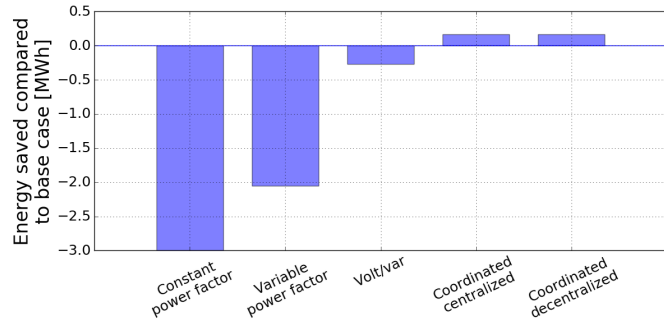


Fig. 6. Comparison of energy saved compared to base case.

proposed ORPD method make use of the entire voltage band. Local RPC schemes that act on local information are not capable of making use of the entire voltage band.

## VI. CONCLUSION

In this paper a zone based coordinated ORPD scheme has been proposed that aims to improve system efficiency by minimizing system losses while ensuring voltage within the prescribed bounds. The proposed method has been implemented in centralized and decentralized manner on the chosen test case. Hierarchical agglomerative clustering has been used to define zones within a radial distribution network with high voltage interdependency. The zone based implementation reduces the dimensions of the optimization problem. This aids in efficient utilization of local reactive power resources. By far the most important advantage of decentralized implementation is that unlike centralized implementation it is scalable. Additionally, it was observed that the performance in terms of selected key performance indices is comparable to centralized implementation.

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