Hybrid Quantum Physics-Informed Neural Networks for Power Flow Analysis

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***Abstract*—In this paper, we develop a hybrid quantum physics-informed neural network (QPINN) for power flow analysis (PF). Two important modifications contribute to the enhanced performance of QPINN: (i) a parameterized quantum circuit is integrated within the neural network architecture, and (ii) a physical loss function is developed based on the PF equations. Experiments are conducted using a small-size dataset based on a 4-bus test system. A linear regression model (LR) and a multilayer perceptron (MLP) are used to perform a systematic performance comparison. The comparison is based on the (i) training error, (ii) validation error, (iii) generalization ability, and (iv) robustness. The results show that QPINN outperforms both LR and MLP and, therefore, can improve deep learning-based PF analysis in the noisy-intermediate-scale quantum (NISQ) era.**

***Index Terms*—Metamodeling, model-guided neural networks, variational quantum algorithms, gate-based quantum computing, distribution networks.**

I. Introduction

Deep learning-based power flow (PF) analysis has been widely used, due to its capability to effectively capture and learn complex nonlinear relationships between inputs and outputs (e.g., [1]). Yet, as the scale and complexity of modern power systems continue to expand, deep learning-based PF analysis faces challenges, particularly in terms of computational efficiency and the ability to generalize effectively. Furthermore, the demand for extensive training data further compounds these challenges, particularly for large-scale modern power systems [2]. Considering a neural network as a composition of different building blocks—including forward propagation, loss calculation, backward propagation, and parameter updates—offers opportunities for enhancing its performance through individual and joint modifications of these building blocks.

The integration of prior physical knowledge into neural networks has demonstrated substantial enhancements in deep learning models across various fields. This integration results in a departure from the "black box" characteristics of neural networks and imparts to them an understanding of the underlying physical model. For PF analysis, the focus is on incorporating prior knowledge, including quality and inequality constraints, the physical model, and network topology, into the neural network, thus rendering it *informed*, *aware*, *guided*, or *model-based* [3]. Such informed neural networks undergo modifications in their forward, loss, and/or backward steps, which leads to different algorithms. For example, a discretized version of the physical model can guide the training process within the loss building block (e.g., [4]). Similarly, in graph neural networks, node and edge information is embedded into the model architecture (e.g., [5], [6]).

Black-box neural networks offer no guarantees about potential constraint violations and can provide infeasible solutions in PF analysis and Optimal Power Flow (OPF) problems (e.g., [7], [8]). They might predict line currents or other physical quantities beyond the limitations and violate inequality constraints. A potential solution to address this concern is to use a sufficiently large dataset that adequately covers the solution space. However, generating such a dataset can be a challenging task in many situations due to the limitations and privacy issues, among others. Furthermore, relying solely on a large dataset can result in overfitting, where the neural network is only useful for the specific scenarios included in the dataset and becomes ineffective when changes occur in the grid topology or operational conditions [3], [9].

In the growing body of work on the use of neural networks for PF analysis, researchers have explored different techniques to meet different types of constraints. For example, to avoid solving equality constraints directly by neural networks, they can predict a part of the variable set, and then a PF solver can be used to complete the variable set. This idea is also known as completion [7], [8]. Another example is incorporating the grid topology information, i.e., the admittance matrix, into the hidden layers [10] and/or the input layer [9] in order to follow the equality and inequality constraints. The grid topology can also be incorporated into a graph neural network [11], [12].

On the other hand, the power systems community has shown considerable interest in leveraging quantum computing paradigms to address different power system applications, including PF analysis, OPF, and unit commitment [13]–[15]. Quantum neural networks (QNN) are an emerging quantum algorithm that exhibits substantial potential in enhancing the efficacy of deep learning models [16]–[18]. They comprise layers of qubits and are trained using pairs of quantum gates. They are, therefore, able to explore high-dimensional feature spaces with circuits of a few qubits and limited depth, potentially leading to superior performance in many machine

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learning tasks, including deep learning-based PF analysis. In addition, the inherent randomness of quantum phenomena allows for the capture of complex relationships between inputs and outputs with reduced reliance on large datasets [19]. These advantages not only enhance the learning process but also address data scarcity concerns, and hence make QNNs a promising candidate for learning from small and/or noisy datasets [20].

In this study, therefore, we employ a parameterized quantum circuit to augment the neural network architecture in the forward building block, while also devising a modified loss function that leverages the physical understanding of power systems within the loss building block. This results in the development of a hybrid quantum physics-informed neural network, hereafter called QPINN, for PF analysis. Experiments are conducted using a 4-bus test system [21], wherein the performance of QPINN is compared with a linear regression model (LR) and a multilayer perceptron (MLP). The quantum component of QPINN is simulated using the statevector simulator provided by Qiskit.

II. Power Flow Analysis

Power flow (PF) analysis is a fundamental task in power system modeling. The aim is to calculate the voltage magnitude and phase angle, the complex voltages, at all buses within the power system. It can be performed based on the AC PF equations, which are a set of nonlinear equations that relate the complex voltage and power at each bus of the power system

$$p_i = \sum_{j=1}^{n} v_i v_j (g_{ij} \cos \delta_{ij} + b_{ij} \sin \delta_{ij}), \quad (1)$$

$$q_i = \sum_{j=1}^{n} v_i v_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}), \quad (2)$$

where i and j denote the indices of the buses, n represents the total number of buses within the power system, v_i and δ_i are the magnitude and phase angle of the complex voltage at bus i, p_i and q_i are the calculated active and reactive power injection/consumption at bus i, respectively. g_{ij} and b_{ij} stand for the real and imaginary parts of the admittance between buses i and j. $\delta_{ij} = \delta_i - \delta_j$ shows the phase angle difference between the complex voltages at buses i and j. Traditionally, the solution to these equations is obtained through the application of numerical techniques, such as the Newton-Raphson method (NR) [22], which iteratively converges towards the solution. The real and imaginary parts of the complex voltage are respectively

$$\mu_i = v_i \cos \delta_i, \quad (3)$$

$$\omega_i = v_i \sin \delta_i. \quad (4)$$

In this paper, $\vec{\mu}$ and $\vec{\omega}$ are used as the output labels to develop a linear regression model (LR), a multilayer perceptron (MLP), and a hybrid quantum physics-informed neural network (QPINN) for PF analysis. The input features include active $\vec{p^d}$ and reactive power consumption $\vec{q^d}$ at load buses.

III. Hybrid Quantum Physics-informed Neural Networks for Power Flow Analysis

A hybrid quantum-classical neural network inspired by our previous work [23] is developed and trained using a modified loss function. The loss function is a combination of a supervised and a physical penalty term. A representation of the resultant QPINN is depicted in Figure 1.

In the *forward* block, the architecture of the neural network includes input, hidden, and output layers, as well as, a parameterized quantum circuit (PQC) sandwiched with two sets of classical hidden layers. The input layer includes the input features, which are given active and reactive power consumption at load buses, i.e., $\vec{p^d} = [p^d_0, p^d_1, \ldots, p^d_n]$ and $\vec{q^d} = [q^d_0, q^d_1, \ldots, q^d_n]$. The output layer consists of the output labels, which are the real and imaginary parts of the complex voltages at all load buses, i.e., $\vec{\mu} = [\mu_0, \mu_1, \ldots, \mu_n]$ and $\vec{\omega} = [\omega_0, \omega_1, \ldots, \omega_n]$. To introduce nonlinearity, the Rectified Linear Unit (ReLU) activation function is employed

$$\sigma(x) = \max(0, x). \quad (5)$$

In each iteration of the training process, data is first encoded from the input layer to a lower dimension using the first set of classical hidden layers, followed by a PQC. The PQC receives data from the immediate hidden layer and, after processing, passes the data to the second set of hidden layers, which decode the data and eventually connect to the output layer. The predicted output labels are then sent to the *loss* block to evaluate the performance of the neural network based on the modified loss function. In the *backward* block, the trainable parameters of the neural network are adjusted based on its performance and then updated in the *update* block. The training process continues until a maximum number of epochs is obtained. The following subsections explain the PQC and the modified loss function used in this work.

A. Parameterized Quantum Circuit

Unlike classical bits that can only take on values of 0 or 1, qubits can exist in a superposition of both states. This means that prior to measurement, a qubit can be in a state that is a linear combination of the basis states 0 and 1. This characteristic of qubits can potentially increase the accuracy of a quantum neural network as it introduces true randomness into the computation.

In this work, we use a parameterized quantum circuit (PQC) to enhance the performance of a classical neural network for PF analysis. The structure includes a feature map, an ansatz, and the measurement. Figure 2 shows the basic structure of a PQC that can be used as a pure quantum neural network or be integrated to a classical neural network, as described in [23]. The data is first encoded into a quantum state using the feature map, which is then processed through the ansatz. The ansatz includes parameterized quantum gates that are trainable. Finally, the measurement is performed to obtain the output of the circuit, which is the probability of being 0 or 1, and needs further post-processing to yield problem-specific results.

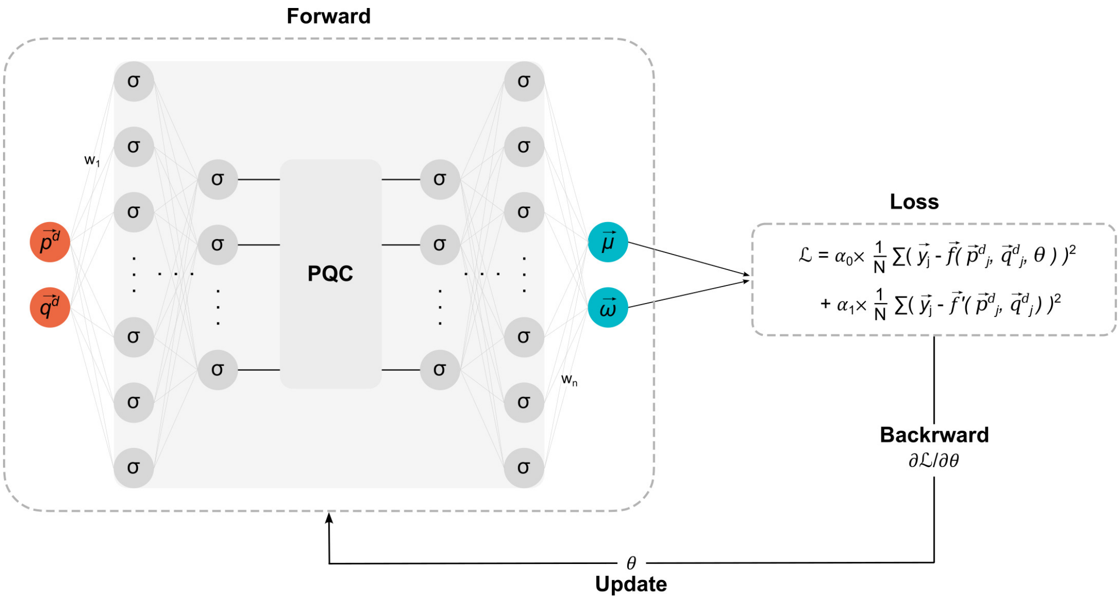

Fig. 1. Schematic depiction of the hybrid quantum physics-informed neural network (QPINN) designed for power flow (PF) analysis with a parameterized quantum circuit (PQC) replacing one hidden layer. The data flow starts from a hidden layer, passes through the PQC, and continues to the subsequent hidden layer after the measurement. The loss function includes supervised and physical penalty terms. The input layer includes the input features, which are given active and reactive power consumption at load buses, i.e., $\vec{p^d} = [p^d_0, p^d_1, \ldots, p^d_n]$ and $\vec{q^d} = [q^d_0, q^d_1, \ldots, q^d_n]$. The output layer consists of the output labels, which are the real and imaginary parts of the complex voltages at all load buses, i.e., $\vec{\mu} = [\mu_0, \mu_1, \ldots, \mu_n]$ and $\vec{\omega} = [\omega_0, \omega_1, \ldots, \omega_n]$.

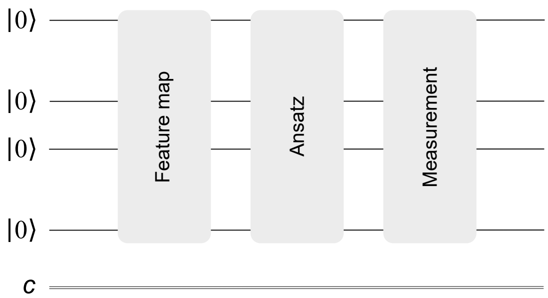

Fig. 2. Schematic of the basic structure of a parameterized quantum circuit (PQC). The PQC receives a number of qubits in the vacuum state. The structure includes a feature map, an ansatz, and the measurement. The feature map and the ansatz consist of quantum gates and change the vacuum state.

B. Modified Loss Function

A modified loss function integrating both supervised and physical penalty terms is developed. The supervised penalty terms is the mean squared error of the output obtained by the neural network compared to the output obtained from NR, defined as

$$MSE_{supervised} = \frac{1}{N} \sum_{j=1}^{m} (\vec{y}_j - \vec{f}(\vec{p}^d_j, \vec{q}^d_j, \theta))^2. \tag{6}$$

Here $\vec{f}(\cdot)$ represents the neural network, $\vec{p}^d_i$ and $\vec{q}^d_i$ are the active and reactive power vectors for data point j. θ is the trainable parameters of the neural network. $\vec{y}_j = \{\vec{\mu}_j, \vec{\omega}_j\}$ is the ground-truth data obtained from NR. N is the total number of data points. The physical penalty term is the mean squared error of the output obtained by the loss function based on the PF equations (1)–(2) and that obtained from NR, defined as

$$MSE_{physical} = \frac{1}{N} \sum_{j=1}^{m} (\vec{y}_j - \vec{f}'(\vec{p}^d_j, \vec{q}^d_j))^2. \tag{7}$$

where $\vec{f}'(\cdot)$ is a derivation of the PF equations. Note that each penalty term has a coefficient, e.g., a_0 and a_1, that specifies the share of the penalty term in the loss function but also ensures that the loss does not surpass a certain limit. Detailed information about the modified loss function is presented in a related work by the authors.

IV. Results

Experiments are carried out based on a modified version of the 4-bus test system [21]. This selection is motivated by the limitations of current quantum hardware, particularly in terms of available qubits and the executable circuit depth. Moreover, the nature of PF analysis aligns well with the chosen systems, as active p^d_i and reactive q^d_i power are typically known for all load buses, e.g., $i = 1, 2, \ldots, n$ while complex voltages $v_i = \mu_i + j\omega_i$ are usually only known for the reference bus, e.g., $i = 0$. Thus, a simplified PQ test system is chosen to maintain computational simplicity.

A. Model Setup

The dataset comprises a total of 512 data points, which are randomly selected from a normal distribution. Input features include active $\vec{p}^d_i$ and reactive $\vec{q}^d_i$ power consumption, while the output labels involve real $\vec{\mu}$ and imaginary parts $\vec{\omega}$ of complex voltages of all buses. Ground-truth data is generated using

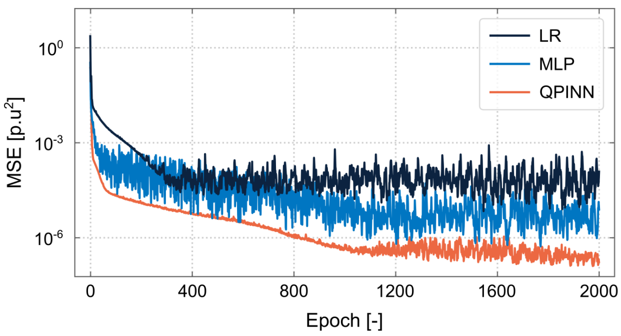

Fig. 3. Performance comparison of the linear regression model (LR), multilayer perceptron (MLP), and hybrid quantum physics-informed neural network (QPINN) based on the average mean squared error (MSE) of the training dataset during the training process for the 4-bus test system. The experiment is performed based on a noisy dataset.

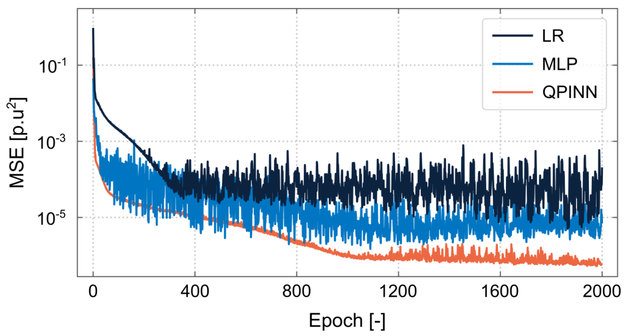

Fig. 4. Performance comparison of the linear regression model (LR), multilayer perceptron (MLP), and hybrid quantum physics-informed neural network (QPINN) based on the average mean squared error (MSE) of the validation dataset during the training process for the 4-bus test system. The experiment is performed based on a noisy dataset.

the Newton-Raphson numerical method (NR). The dataset is divided into three subsets, allocating 25% for training, 25% for validation, and the remaining 50% for testing. The training dataset intentionally includes only 128 data points to highlight the enhanced performance of the developed QNNs. The training process concludes after 2000 epochs. The utilized loss function for the LR and MLP is the supervised MSE described in Equation (6), and for QPINN, it is a combination of Equations (6) and (7). The activation function is ReLU, and the Adam optimization algorithm is employed for training.

B. Model Performance

The performance of a linear regression model (LR), a multilayer perceptron (MLP), and the hybrid quantum physics-informed neural network (QPINN) is systematically evaluated based on a noisy dataset, and the role of quantum computing in enhancing the performance of deep learning-based PF is explored. The assessment is based on the (i) training error, (ii) validation error, (iii) generalization ability, and (iv) robustness. The results are provided in the following:

1) Training error: The training error obtained by LR, MLP, and QPINN is compared in Figure 3 based on the average MSE obtained for the training dataset over 2000 epochs. Accordingly, the training error obtained by QPINN after 2000 epochs is respectively 71% and 54% lower than LR and MLP.

2) Validation error: The validation error obtained by LR, MLP, and QPINN is compared in Figure 4 based on the average MSE obtained for the validation dataset over 2000 epochs. Accordingly, the validation error obtained by QPINN after 2000 epochs is 85% and 55% lower than LR and MLP, respectively.

3) Generalization ability: Different data points beyond the training dataset, i.e., the testing dataset, are considered. The best testing error obtained by QPINN over 2000 epochs is lower than LR and MLP by 72% and 48%, respectively. In addition, the best testing error obtained by LR, MLP, and QPINN is compared in Table I based on the average MSE obtained for the voltage magnitude $[V^2]$, phase angle $[rad^2]$, and line current $[A^2]$ for the testing dataset over 2000 epochs.

TABLE I
PERFORMANCE COMPARISON OF THE LR, MLP, AND QPINN BASED ON THE TESTING MSE OBTAINED FOR VOLTAGE MAGNITUDE $[V^2]$, PHASE ANGLE $[rad^2]$, AND LINE CURRENT $[A^2]$. THE EXPERIMENT IS PERFORMED BASED ON A NOISY DATASET.

Algorithm	MSE $[V^2]$	MSE $[rad^2]$	MSE $[A^2]$
QPINN	1.11×10^{-5}	7.46×10^{-7}	1.14×10^{-5}
MLP	5.70×10^{-3}	3.72×10^{-7}	2.80×10^{-5}
LR	2.52×10^{-2}	4.70×10^{-6}	4.04×10^{-5}

Note that the voltage magnitude, phase angle, and line current are calculated based on the predicted $\vec{\mu}$ and $\vec{\omega}$. Accordingly, QPINN outperforms both LR and MLP in terms of the MSE obtained for the voltage magnitude and line current.

4) Robustness: The impact of noisy data is investigated by systematically introducing controlled levels of noise, ranging from 0% to 10%, to the dataset. Specifically, this means that for a given level of noise, the corresponding percentage of the total dataset is altered by random modifications. The investigation is based on the average MSE obtained for the testing dataset. The results shown in Figure 5 suggest QPINN exhibits a greater degree of robustness against noisy data compared to MLP. It can be seen that for the training and testing datasets without noises, MLP outperforms QPINN. However, the performance of MLP is highly affected by up to 54% as the noise level increases.

V. CONCLUSION

This study presents the development of a hybrid quantum physics-informed neural network (QPINN) for power flow (PF) analysis. QPINN introduces two crucial enhancements contributing to its superior performance: (i) integration of a parameterized quantum circuit (PQC) within the neural network architecture, and (ii) formulation of a modified loss function grounded on the PF equations.

Experimental validation is conducted using a small-scale dataset derived from a 4-bus test system. To systematically evaluate its efficacy, QPINN is benchmarked against two established models: a linear regression model (LR) and a

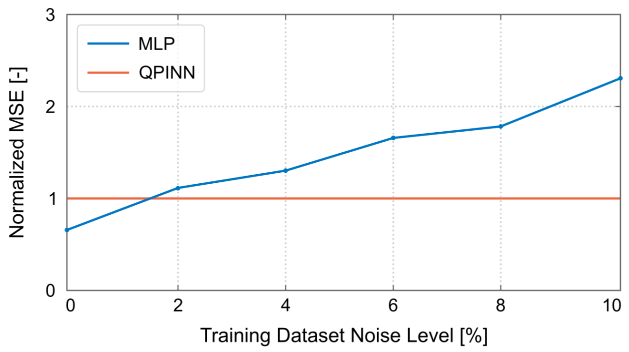

Fig. 5. Performance comparison of the multilayer perceptron (MLP) and hybrid quantum physics-informed neural network (QPINN) based on the testing MSE under varying levels of noise in the training dataset for the 4-bus test system. The MSE values are normalized relative to the testing MSE obtained for the QPINN at the noise level of 10%.

multilayer perceptron (MLP). The comparison is based on assessment criteria including (i) training error, (ii) validation error, (iii) generalization ability, and (iv) robustness.

The findings demonstrate that QPINN surpasses both LR and MLP across all evaluation metrics. In terms of training, validation, and testing errors, QPINN respectively performs up to 71% and 48% better than MLP and LR. In addition, QPINN is more robust by up to 54% against noisy data compared to MLP. Thus, QPINN exhibits promise for enhancing deep learning-based power flow analysis, particularly in the context of the noisy-intermediate-scale quantum (NISQ) era.

Several areas remain that need to be addressed in future work, including proving the scalability of the proposed deep learning model both in the short term and long term, reducing the computational burden and complexities, and comparing the proposed model with more advanced and state-of-the-art models from the literature.

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