Distributed cooperation for voltage regulation in future distribution networks

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Abstract—As the penetration of distributed energy resources (DERs) increases significantly in the distribution networks, their integration reshapes distribution system power flows and leads to serious voltage quality problems in distribution networks. In this paper, a novel distributed cooperation voltage regulation method is proposed for future distribution networks. The proposed method can minimize the number of agents involved in the voltage regulation and it minimizes the change of required power for voltage regulation, which together minimizes the need for re-dispatching, i.e. the impact of voltage regulation on exchange of energy. Moreover, the proposed method performs online optimization, i.e. the value of the decision variable is physically implemented as a controller set-point at each iteration which reduces the response time. The presented algorithm is benchmarked against ADMM and centralized optimization. The results show the voltage regulation effectiveness. Compared to the centralized method, the proposed method performs better in the case of a single agent failure, and compared to the ADMM method, the proposed method greatly reduces the action time and does not require retuning if network topology or agent participation changes.

Index Terms—voltage regulation, distribution networks, distributed energy resources, voltage sensitivity, distributed control

I. INTRODUCTION

In recent years, a significant increase in the penetration of distributed energy resources (DERs) such as distributed generators, battery energy storage, and plug-in electric vehicles (PEVs) has been witnessed. Along with the advantages, many challenges in the control and operation of power grid arise with the change of end-consumer characteristics in distribution networks, due to the intermittent nature of DERs. Among them, voltage management became an urgent issue because the conventional Voltage/VAR control devices such as shunt capacitor banks cannot respond well to the voltage deviation problem in the future distribution networks [1].

Fortunately, with the development of modern information and communication technologies (ICT), new opportunities are created to deal with the voltage problem through controlling and accommodating the DERs. Besides, the distribution system operators (DSOs) are willing to incentive the consumers with DERs to actively provide ancillary services [2][3]. For instance, the UK has been working on Active Network Management (ANM) to increase renewable energy capacity [4].

As for voltage management, consumers are also encouraged to respond actively. For example, according to the German VDE application regulation VDE-AR-N 4105 in 2011, photovoltaics (PVs) must be capable of providing reactive power for voltage regulation [5]. An Australian standard for grid-connected inverters of grid-connected PVs (AS4777) specifies how they should respond to high or low grid voltages [6]. Thus, there is no doubt that the consumers will be encouraged to take the voltage regulation tasks in the future.

The ANM approach to controlling and coordinating active units to realize voltage regulation can be divided into three categories, namely centralized, decentralized, and distributed methods [7]. In all of these approaches, the agents must be willing to install smart meters to monitor their information and share resources. The centralized method is commonly realized through solving the optimal voltage regulation problem, which requires a central coordinator with access to all information of the network [8][9]. The main drawback of the centralized control is the computational cost to handle a big amount of data, which includes the investments in communication channels. Moreover, the data privacy, cyber-attacks and communication failure need to be carefully considered when the centralized method is applied. In response, decentralized control methods appeared which divide the network into several zones allowing to apply the optimal method in a much smaller scale in each zone [10][11]. This approach creates new challenges. First, it is still unclear how to partition the network into zones to realize the global optimal voltage regulation. Second, the simultaneous responses of different zones will interact with each other and cause operational conflicts [12][13]. Thus, the solutions based on distributed algorithms are gradually becoming more attractive.

The control scheme of the distributed cooperation approach is shown in Figure 1, in which communication channels are aligned with cables to realize peer-to-peer information exchange. The Alternating Direction Method of Multipliers (ADMM), an algorithm that blends the benefits of Dual Decomposition algorithm (DD) and augmented Lagrangian methods, is widely used to solve distributed convex optimization problems including voltage regulation [14][15]. This method, and many other distributed optimization methods, decompose the global optimization problem into a small local sub-problems. The convergence speed and results accuracy are most important challenges addressed in the literature [16][17]. References [18][20] extend the ADMM method to unbalanced three-phase networks, demonstrating successful convergence even for larger networks. In order to realize fairness among agents, consensus-based algorithms are used in references
in order to maintain an equal power-sharing among the resources. However, voltage violation is a local problem, and hence, some agents have higher potential for voltage regulation than the others. In this light, fairness in power-sharing is sub-optimal from the purely technical point of view.

Although the ADMM algorithm is one of the most known distributed approaches, it still requires significant computational time to reach convergence for solving the power flow problem [19]. Moreover, the choice of Lagrange multipliers and penalty parameters plays a vital role in obtaining realistic convergence times [18]. Since some applications, such as voltage regulation, require a relatively fast response (in the order of seconds to minutes), the short convergence time and fast response is of essence. Therefore, reference [24] based on dual ascent algorithm proposes an online optimization for distributed voltage control, in which set-points are implemented after each iteration.

To get the benefits of distributed control and overcome the above shortcomings, a novel distributed cooperation method is proposed. The main contribution of this paper is twofold. First, the proposed method uses the minimal number of agents and the minimal amount of contributed power to regulate voltage. By reducing the number of agents and their power contribution, we minimize the need for re-dispatching, i.e. the deviations from the nominal power exchange are minimized. The nominal power exchange is associated with comfort levels and financial transactions, and hence it is essential to minimize deviations from it. The proposed algorithm relies on voltage sensitivity to engage only the agents with the highest potential to regulate voltage. Second, the proposed method implements the set-points in each iteration, in the spirit of online optimization. Hence, the response time of the algorithm is minimal and the voltage violations are acted upon quickly, much faster than with ADMM and similar algorithms.

The voltage problem formulation is introduced in Section II. In Section III we presented the proposed distributed cooperation voltage regulation method. The voltage sensitivity characteristic and the process of applying the method in online fashion is also presented in this section. In Section IV cases studies are shown and concluding remarks are presented in Section V.

II. VOLTAGE PROBLEM FORMULATION

Each Node is monitored and controlled by an Intelligent Electronic Device (IED) which is defined as one agent in the proposed distributed cooperation method. The agent can measure the data at its own location, exchange its information with neighbors, do simple calculations, provide its flexibility information (i.e. its available capacity for regulation) and control its own controllable assets to change their absorbed or injected power. In this paper, the controllable assets are denoted as Distributed Energy Resources (DERs) and involve plug-in EVs which can charge or discharge, PVs which can change their generated active power, capacitor banks which can provide reactive power and so on. Because the time-series characteristics of those elements very significantly, and the energy capacity, SOC and even EV user behavior can be considered internal to the assets and privacy protected, in this paper we do not include their models in the voltage regulation algorithm. Instead available power limits are assumed given and deterministic. In this section, after the introduction of basic terminology, the centralized voltage regulation model is presented. This model is used as a benchmark for the distributed approach.

A. Power flow formulation

For radial distribution networks with a discrete set of loads, branch flow model is well established based on DistFlow equations [25] as:

\[
P_{i-1} = P_i + r_i \frac{P_i^2 + Q_i^2}{v_i^2} + p_{i-1},
\]

\[
Q_{i-1} = Q_i + x_i \frac{P_i^2 + Q_i^2}{v_i^2} + q_{i-1},
\]

\[
v_{i+1}^2 - v_i^2 = -2 (r_i P_i + x_i q_i) + (v_i^2 + x_i^2) \frac{P_i^2 + Q_i^2}{v_i^2},
\]

where \( i = 1, 2, ..., n - 1 \) enumerates DERs agents of the feeder sequentially connected in a line, \( P_i \) and \( Q_i \) are the active power and reactive power from agent \( i - 1 \) to agent \( i \). The positive value of \( P_i \) and \( Q_i \) represents that the power is transferred from agent \( i - 1 \) to agent \( i \), while if the power is transferred from agent \( i \) to agent \( i - 1 \), the value of \( P_i \) and \( Q_i \) are negative. \( v_i \) represents the voltage at agent \( i \), and \( r_i \) and \( x_i \) are separately the resistance and reactants of the line \( i \).
\( q_i \) describe the net demand of agent \( i \) which are composed of local consumption \( (p_i^c \text{ and } q_i^c) \) minus local generation \( (p_i^g \text{ and } q_i^g) \) and power adjustment \( (p_i^a \text{ and } q_i^a) \) controlled by agent, i.e.,
\[
p_i = p_i^c - p_i^g - p_i^a, \quad q_i = q_i^c - q_i^g - q_i^a. \tag{4}
\]

The positive value of \( p_i^a \) and \( q_i^a \) is the increased injection power from assets of agent \( i \) into node \( i \). On the contrary, the negative value of \( p_i^a \) and \( q_i^a \) means the power is transferred from the node to the assets of the agent.

Finite feeder line with the tap changing transformer in the beginning can be modeled by the boundary conditions in the rescaled p.u. variables:
\[
v_0 = 1, \quad p_n = q_n = 0. \tag{5}
\]

\( B. \) Voltage deviation model

The voltage deviation between node \( i \) and node \( i + 1 \) can be expressed as:
\[
\Delta v_i = v_i - v_{i-1}
\tag{6}
\]
\[
= I_v (r_i + j x_i)
\tag{6}
\]
\[
= \frac{P_i}{v_i} (r_i + j x_i)
\tag{6}
\]
\[
= \frac{P_i r_i + x_i Q_i}{v_i} + j \frac{P_i x_i - r_i Q_i}{v_i}.
\tag{6}
\]

As the voltage difference angle between two nodes is small in distribution networks, the voltage deviation across the feeders is approximately equal to the real part of the voltage drop. Therefore, the voltage deviation \( (\Delta v_i) \) can be approximated as:
\[
\Delta v_{i-1} \approx \frac{P_i r_i + x_i Q_i}{v_i}. \tag{7}
\]

Then the voltage at node \( i \) can be expressed as:
\[
v_i = v_0 - \sum_{k=1}^{i} (v_{k-1} - v_k)
\tag{8}
\]
\[
= v_0 - \sum_{k=1}^{i} \frac{P_k r_k + x_k Q_k}{v_k}.
\tag{8}
\]

It is obvious that the voltage at node \( i \) depends on the injected and absorbed power of all the nodes in the network. As the resistances and the reactance of the lines are constant values and the voltage at node \( i \) \( (v_i) \) depends on the power injected or absorbed in the networks, Equation (8) can be expressed as:
\[
v_i = f (p_1, p_2, ..., p_n, q_1, q_2, ..., q_n). \tag{9}
\]

From the above analysis, the increment of generated power from the agent will automatically lead to the voltage rise problem, and the decrements of generated power from DERs will lead to the voltage drop problem. Thus the voltage deviation problem will become much heavier with increasing installation of DERs. Moreover, due to the intermittent characteristic of DERs, future distribution networks will face the voltage deviation problem more often. Fortunately, if the negative reactive power is applied or the load is sufficiently flexible to be increased, the voltage rise problem can be reduced. Vice versa, the voltage drop problem can be solved by applying positive reactive power or reducing consumption. That means if we can make good use of DERs, their high penetration would not have a bad influence on the voltage stability, but can instead be utilized to solve the voltage problem.

\( C. \) Voltage regulation model

In the centralized voltage regulation approach, the main objective is to lower the use of power to regulate the voltage, while staying within desired limits.
\[
\min_{p_i^a, q_i^a} \sum_{i=1}^{n-1} (\|p_i^a\| + \varphi \|q_i^a\|), \tag{10}
\]

In Equation (10), \( \varphi > 0 \) represents the price ratio of reactive power and active power for voltage regulation, i.e., if the price to provide reactive power for voltage regulation is lower than active power, then the value of \( \varphi \) will be lower than 1. The term \( \varphi \) shows preference for using active or reactive power. To ease notation and without loss of generality, we drop this term from further derivations by setting its value to one. The agent operation set-point should be constrained by the room of controllable power as:
\[
p_i^a \leq p_i^a \leq \overline{p}_i, \forall i, \tag{11}
\]
\[
q_i^a \leq q_i^a \leq \overline{q}_i, \forall i, \tag{12}
\]

where \( \overline{p}_i \) and \( \overline{q}_i \) are negative and represent the highest controllable active and reactive power that can be absorbed by agent \( i \) to regulate the voltage, while \( \overline{p}_i \) and \( \overline{q}_i \) are positive values that represent the highest controllable active and reactive power that can be generated by agent \( i \).

The power flow equations are represented by DistFlow model as Equations (1)-(5) in subsection II-A.

The bus voltage magnitudes should be maintained within the acceptable range as:
\[
u \leq v_i \leq \overline{v}, \forall i, \tag{13}
\]
where \( \underline{v} \) is the voltage lower limitation, and \( \overline{v} \) is the voltage upper limitation, according to voltage violation standard of the distribution grid.

\( D. \) Linearized voltage problem

The optimal voltage regulation model given by Equations (1)-(5) and (10)-(13) is non convex because of the flow related constraints (1)-(5). Therefore, the voltage regulation model must be linearized in order to solve the regulation problem. In this paper, Equation (3) is linearized as:
\[
v_i = v_i^b + v_i^a, \tag{14}
\]
\[
v_i^a = \sum_{j=1}^{n-1} \left( \frac{\partial v_i}{\partial p_j} p_j^a + \frac{\partial v_i}{\partial q_j} q_j^a \right), \tag{15}
\]
where \( v_i^0 \) is the voltage at agent \( i \) before voltage regulation, \( v_i^p \) is the difference in the voltage value caused by voltage regulation. The voltage sensitivities \( \frac{\partial v_i}{\partial P_i} \) and \( \frac{\partial v_i}{\partial Q_i} \) show the voltage regulation potential of agents. The higher voltage sensitivity of an agent, the higher its voltage regulation potential and lower is the power needed to perform regulation.

In this paper, Perturb and Observe Power Flow Based method [26] is applied to get the voltage sensitivity. The voltage sensitivity to active power \( s^p_{ij} \) is defined as the deflection per unit voltage at node \( i \) across the active power at node \( j \) (\( \frac{\partial v_i}{\partial P_j} \)), and \( s^q_{ij} \) is defined as \( \frac{\partial v_i}{\partial q_j} \). If the voltage sensitivity \( s^p_{ij} \) is needed, the calculation steps of the Perturb and Observe Power Flow Based method are as follows,

1) Record the voltage \( v_i \).
2) Add a small active power injection \( \Delta P_j \) at node \( j \).
3) Record the new voltage \( v_i \), and get the difference \( \Delta v_i \) through subtracting the voltage \( v_i \) (\( \Delta v_i = v_i - v_i \)).
4) \( \Delta v_i \) is divided by \( \Delta P_j \) to obtain voltage sensitivity \( s^p_{ij} \) (\( \frac{\partial v_i}{\partial P_j} \)).

Finally, the linear formulation of the voltage regulation model is given as:

\[
\begin{align*}
\min_{p^0, q^0} & \sum_{i=1}^{n-1} (\|p_i^0\| + \|q_i^0\|), \\
\text{s.t.} & \quad p_i^0 \leq P_i^0 \leq \bar{P}_i, \forall i, \\
& \quad q_i^0 \leq Q_i^0 \leq \bar{Q}_i, \forall i, \\
& \quad \bar{v} \leq v_i \leq \bar{v}, \forall i, \\
& \quad v_i = v_i^0 + \sum_{j=1}^{n-1} (s^p_{ij}p^0_j + s^q_{ij}q^0_j). 
\end{align*}
\] (16)

According to the Perturb and Observe Power Flow Based method, the values of voltage sensitivities \( s^p_{ij} \) and \( s^q_{ij} \) can be seen as known time-varying constants, and the only unknown variables are \( p_i^0 \) and \( q_i^0 \). Therefore, the voltage regulation problem becomes linear and convex.

The linearized and centralized voltage regulation from (16)-(20) is used as a benchmark for the distributed approach, which is described in the following section.

III. DISTRIBUTED COOPERATION FOR OPTIMAL VOLTAGE REGULATION

In the distributed approach, beside the effort to minimize the contribution of active and reactive power to regulate the voltage, we also try to minimize the number of engaged agents. To do this properly, we dispatch the agents according to their capacity to regulate, which is derived based on voltage sensitivity.

A. Voltage sensitivity characteristic

The voltage sensitivity of node \( i \) to node \( i+1 \) (\( \frac{\partial v_i}{\partial P_{i+1}} \)) can be shown as:

\[
\frac{\partial v_i}{\partial P_{i+1}} = \left( \frac{\partial v_i}{\partial P_i} \right) \left( \frac{\partial P_i}{\partial P_{i+1}} \right) \left( \frac{\partial P_{i+1}}{\partial P_{i+1}} \right). \tag{21}
\]

For an arbitrary agent (node \( i \)) in radial distribution networks, if only \( p_i \) changes, the increment of \( P_i \) is approximately equal to the increment of \( p_i \), namely \( \frac{\partial P_i}{\partial p_i} \approx 1 \), then \( \frac{\partial v_i}{\partial P_{i+1}} \) can be derived as:

\[
\frac{\partial v_i}{\partial P_{i+1}} = \frac{\partial v_i}{\partial P_i} \cdot \frac{\partial P_i}{\partial P_{i+1}}. \tag{22}
\]

Based on Equation (1), the \( \frac{\partial P_{i+1}}{\partial P_{i}} \) can be derived as follows:

\[
\frac{\partial P_{i+1}}{\partial P_i} = 1 + \frac{2r_i}{v_i^3} P_i^2 + \frac{Q_i^2}{v_i^3} \frac{\partial P_i}{\partial P_i} + 2r_i v_i^2 P_i v_i. \tag{23}
\]

If nodes \( i-1, i \) and \( i+1 \) extract power, then \( p_{i-1}, p_i \), and \( p_{i+1} \) in Figure 2 are positive. The transferred power over lines \( i-1, i \) and \( i+1 \) (\( P_{i-1}, P_i, P_{i+1} \)) is positive, and the relationship of voltage from node \( i-1 \) to node \( i+1 \) is \( v_i \geq v_i \geq v_i+1 \). Since the value of \( \frac{\partial P_i}{\partial P_i} \) is negative and the value of \( P_i \) is positive, the value of \( \frac{\partial P_{i+1}}{\partial P_i} \) is larger than 1.

\[
\frac{\partial P_{i+1}}{\partial P_i} > 1. \tag{24}
\]

Based on Equation (24), the value of \( \frac{\partial P_i}{\partial P_{i+1}} \) is also bigger than 1, and \( \left| \frac{\partial v_i}{\partial P_{i+1}} \right| > \left| \frac{\partial v_i}{\partial P_i} \right| \) in Equation (22). Evidently, in the case of a voltage drop, voltage regulation can be achieved with less resources (i.e. power) if we perform regulation on the neighboring node with lower voltage.

The characteristic of voltage sensitivity \( \frac{\partial v_i}{\partial P_{i+1}} \) can be derived similarly as for \( \frac{\partial v_i}{\partial P_{i+1}} \), and it is shown as:

\[
\frac{\partial v_i}{\partial P_{i+1}} = \left( \frac{\partial v_i}{\partial P_i} \right) \left( \frac{\partial P_i}{\partial P_{i+1}} \right) \left( \frac{\partial P_{i+1}}{\partial P_{i+1}} \right). \tag{25}
\]

Based on Equation (24), the value of \( \frac{\partial P_i}{\partial P_{i+1}} \) is smaller than 1, and \( \left| \frac{\partial v_i}{\partial P_{i+1}} \right| < \left| \frac{\partial v_i}{\partial P_i} \right| \). It is easy to show that \( \left| \frac{\partial v_i}{\partial P_{i+1}} \right| < \left| \frac{\partial v_i}{\partial P_i} \right| \). If we consider reactive power, following the same steps we arrive at a similar result for voltage sensitivity to reactive power. Therefore, Proposition 1 is presented.

Proposition 1. In the case of a voltage drop, the characteristic of voltage sensitivity is as follows:

\[
\begin{align*}
\left| \frac{\partial V_i}{\partial P_{i-1}} \right| \quad & < \left| \frac{\partial V_i}{\partial P_i} \right| \quad < \left| \frac{\partial V_i}{\partial P_{i+1}} \right|, \\
\left| \frac{\partial V_i}{\partial Q_{i-1}} \right| \quad & < \left| \frac{\partial V_i}{\partial Q_i} \right| \quad < \left| \frac{\partial V_i}{\partial Q_{i+1}} \right|. 
\end{align*}
\] (26)

In words, if the voltage drop happens at node \( i \), it is more beneficial to adjust the power of the neighboring node with lower voltage.

Now we consider the case in which the nodes \( i-1, i \) and \( i+1 \) inject power, and \( p_{i-1}, p_i, \) and \( p_{i+1} \) are negative. The power in lines \( P_{i-1}, P_i, P_{i+1} \) is negative, and the respective voltages are \( v_{i+1} \geq v_i \geq v_{i-1} \). Since the value of \( \frac{\partial v_i}{\partial P_i} \) is positive and the value of \( P_i \) is negative, estimating the range of values for \( \frac{\partial v_i}{\partial P_i} \) from Equation (23) is not as straightforward. However, if the increment in line losses is negligible, the value
of $\frac{\partial p_{i,t}}{\partial p_{i,t}}$ is equal to 1, and $\frac{\partial v_i}{\partial p_{i,t+1}} \approx \frac{\partial v_i}{\partial p_{i,t}}$. The higher the voltage, the lower are the line losses, and the value of $\frac{\partial p_{i,t+1}}{\partial p_{i,t}}$ is closer to 1 and the value of $\frac{\partial p_{i,t}}{\partial p_{i,t}}$ is closer to $\frac{\partial v_i}{\partial p_{i,t}}$. In the case of reactive power, following the same steps we reach at the same conclusions for sensitivity. Therefore, Proposition 2 is presented.

**Proposition 2.** In the case of voltage rise, the characteristic of voltage sensitivity is:

$$
\begin{align*}
\left\{ \frac{\partial v_i}{\partial p_{i,t}} \right\} \approx & \left\{ \frac{\partial v_i}{\partial q_{i,t}} \right\} = \left\{ \frac{\partial v_i}{\partial q_{i,t+1}} \right\},
\end{align*}
$$

(27)

In words, if the voltage rise problem happens at node $i$, there is a relatively small difference in adjusting the power by node $i$ itself or by its neighboring nodes with higher voltage.

**B. The process of online distributed cooperation for voltage regulation**

In subsection III-A, we arrived at the conclusion that to mitigate voltage rise, it is almost equally optimal if we regulate the voltage on the node that faces voltage rise or on one of its neighbors. And in the case of the voltage drop, it is much more sensible to regulate it on one of the neighbors. Moreover, in order to guarantee that the distribution network operates well, the agent who faces severe voltage problem should be regulated first. Therefore the agent $i$ with the most severe voltage problem is chosen and given the priority for regulation. In this paper, we define the set $NC_i$ as the union of agent $i$ and its nearest neighbors with voltage regulation ability. If the voltage cannot be regulated only by agents $NC_i$, all the available power of these agents will be used up. On the other hand, if the voltage problem can be regulated only by agents $NC_i$, the power requirement for each of them will be determined.

The evaluation of agents $NC_i$ voltage regulation ability is based on Equation (28). If the voltage of agent $i$ was lower than the voltage limit $v$ and at the same time the maximum voltage regulation capability by agents $NC_i$ ($\sum_{j \in NC_i} (\frac{\partial v_i}{\partial p_{i,t}} p_{j,t}^q + \frac{\partial v_i}{\partial q_{j,t}} q_{j,t}^p)$) is higher than the voltage deviation from the limit ($|v - v_i|$), agents $NC_i$ can regulate the voltage drop problem by themselves. Similarly, if the voltage of agent $i$ was higher than the voltage limit $\overline{v}$ and at the same time the maximum voltage regulation capability by agents $NC_i$ ($\sum_{j \in NC_i} (\frac{\partial v_i}{\partial p_{i,t}} p_{j,t}^q + \frac{\partial v_i}{\partial q_{j,t}} q_{j,t}^p)$) is higher than the voltage deviation from the limit ($|\overline{v} - v_i|$), the voltage rise problem can also be regulated only by agents $NC_i$.

$$
\begin{align*}
|v - v_i| & \leq \sum_{j \in NC_i} \left( \frac{\partial v_i}{\partial p_{i,t}} p_{j,t}^q + \frac{\partial v_i}{\partial q_{j,t}} q_{j,t}^p \right) \quad \text{and} \quad v_i \leq v,
\end{align*}
$$

(28)

$$
\begin{align*}
|\overline{v} - v_i| & \leq \sum_{j \in NC_i} \left( \frac{\partial v_i}{\partial p_{i,t}} p_{j,t}^q + \frac{\partial v_i}{\partial q_{j,t}} q_{j,t}^p \right) \quad \text{and} \quad v_i \geq \overline{v}.
\end{align*}
$$

In order to use the minimal power to regulate the voltage when voltage regulation ability of agent $NC_i$ is sufficient, the optimal voltage regulation model in subsection II-D is applied in distributed and online fashion among the neighbours to determine the regulation setpoint of each node. The online distributed optimal model can be formulated based on Equations (16)-(20) as:

$$
\begin{align*}
\min_{p_{j,t}^q, q_{j,t}^p} \sum_{j \in NC_i,t} \left( ||p_{j,t}^q|| + ||q_{j,t}^p|| \right),
\end{align*}
$$

(29)

s.t. $\begin{align*}
& p_{j,t}^q \leq p_{j,t}^q \leq p_{j,t}^q, j \in NC_i,t, \\
& q_{j,t}^p \leq q_{j,t}^p \leq q_{j,t}^p, i \in NC_i,t, \\
& v \leq v_{i,t+1} \leq \overline{v}, i \in NC_i,t, \\
& v_{i,t+1} = v_{i,t}^b + \sum_{j \in NC_i,t} \left( s_{i,t}^p j \frac{p_{j,t}^q}{s_{i,t}^p} + s_{i,t}^q j \frac{q_{j,t}^p}{s_{i,t}^q} \right),
\end{align*}
$$

(30)

where $p_{j,t}^q, q_{j,t}^p, p_{j,t}^q, p_{j,t}^q, \overline{p}_{j,t}^q, \overline{q}_{j,t}^p, s_{i,t}^p, s_{i,t}^q$ of agent $j$ at time $t$ separately, and $NC_i,t$ is the $NC_i$ of agent $i$ at time $t$, and $v_{i,t+1}$ is the regulated voltage at time $t + 1$.

To relax the requirement for continuous real-time information exchange among agents, an event-triggered communication approach is used. If the agent does not face any voltage problem and does not get any request from others, then there is no need to transfer its information to its neighbor and do any actions. The process of distributed cooperation for optimal voltage regulation is as follows,

1) All agents check if they face the voltage problem or get information exchange request from others or voltage regulation requirements from neighbors. If the agent faces the voltage problem, go to the next step. If the agent gets information exchange request from others, it will exchange the information. If the agent gets voltage regulation order from neighbors, do the voltage regulation action as orders. Otherwise, the agent does not act.

2) Exchange the information ($v_i$) with its neighbours and find the agents who face the most severe problem, i.e agents $i$. Agents $i$ with most severe problem can be more than one in the network because there may be several feeders in a network.

3) Agent $i$ finds the agent set $NC_{i,t}$ to regulate the voltage.

4) If $NC_{i,t}$ is empty, the voltage problem cannot be regulated, and the process stops. If $NC_{i,t}$ is not empty, go to the next step.

5) Calculate the voltage sensitivity ($\frac{\partial v_i}{\partial p_{i,t}}, j \in NC_{i,t}$ and $\frac{\partial v_i}{\partial q_{j,t}}, j \in NC_{i,t}$) using the Perturb and Observe Power Flow Based method in subsection II-D.

6) Judge the agents $NC_{i,t}$ voltage regulation ability based on Equation (28). If the condition (28) is satisfied then nodes $NC_{i,t}$ can regulate the voltage at agent $i$ within the limits and we can proceed to step 7; otherwise, go to step 8.

7) Calculate the power agents $NC_{i,t}$ need to provide ($\frac{p_{j,t}^q, q_{j,t}^p}{s_{i,t}^p}$) based on Equation (29).

8) Agents $NC_{i,t}$ provide all the power they can provide to regulate the voltage.
Algorithm 1: How to find the agents with the locally most severe voltage problem

Result: The Agents $i$ with the locally most severe voltage problem

Initialization;
if $\forall \leq v_i$ then
while $v_i \leq v_j, j \in NC_i$ do
  $i \leftarrow j$;
end
end
if $v_i \leq v_j$, then
while $v_j \leq v_i, j \in NC_i$ do
  $i \leftarrow j$;
end
end

The flow chart of the distributed cooperation for voltage regulation is shown in Figure 3. The voltage is regulated firstly from the agents which face the worst voltage problem, and then through peer-to-peer communication, the distribution network gradually realize self-organized voltage regulation actions. The voltage regulation support searching and information spreading will not stop until the voltage is regulated within the permitted limits or there are no resources available for voltage regulation. If there are sufficient resources, through continuous support from neighbors, the voltage will finally be regulated to an acceptable value. In order to handle singular situations, if one agent receives different voltage regulation instructions, then it will provide the average requested power.

The information exchange scheme based on agent $i$ of the proposed method is shown in Figure 4. The algorithm assumes that the connectivity of agents follows the topology of the distribution network. If one of the agents falls out of the network, or if one of the nodes does not contain an agent, it is assumed that the existing agents are capable of reconnecting and bypassing the troubling node. Therefore, for the distribution grids, the proposed method can tolerate the failure of a single agent.

C. Convergence analysis

The voltage problem shown in Equation (29) is a convex problem with linear inequality constraints and can be written more compactly as

$$\min_{X} C^T M, \quad \text{s.t. } AM \leq B,$$

where $M$ is the vector of decision variables collecting $p_i^a$ and $q_i^a$. $A$ is a coefficient matrix, $C$ and $B$ are coefficient vectors. The Lagrangian equation of this voltage problem is

$$L(X, \lambda) = C^T X + \lambda (AM - B).$$

The voltage sensitivity $S$, which is equal to $\partial V / \partial X$, is in the gradient of the Lagrangian function. The update rules of $M$ are from the higher gradient to the lower gradient, proving that the voltage can be asymptotically regulated within the limits.

IV. Simulation

A. Test grid

To demonstrate the effectiveness of the proposed method, case studies are carried out at the CIGRE low voltage (LV) distribution residential sub-network and the topology is shown in Figure 5 [27]. There are 18 load nodes which host controllable elements to regulate the voltage, and they all can be controlled by the node agents. The base voltage in this test grid is 0.4kV. The agent power of each node is comprised of PVs, plug-in EVs and typical residential load. The PVs model is NA-V135h1 in DIgSILENT, while the load power and plug-
in EVs follow reference [28]. According to this reference, the energy consumption of all plug-in EVs and energy generation of all PVs is 5.52% and 20.81% of the total load in the CIGRE LV network. These total values are equally distributed across the nodes. The EVs, PVs, pure residential load and net active power demand at each node in a typical day is shown in Figure 6.

![Figure 6. CIGRE LV distribution residential sub network](image)

The data of power consumption and power flow losses among these objectives are shown in Table I and the voltage profiles are shown in Figure 7. From Table I we can easily see that the power flow losses in the network do not show a considerable difference between the three objectives, while the power usage for voltage regulation shows a significant difference between them. Minimization of contributing power changes the power contribution by 45.77% of that of the other two objectives. From Figure 7 we see that all of these three objectives regulate the voltage within the limitation. Therefore, when minimizing contributing power to regulate the voltage, we not only keep the voltage within limits, but also achieve this at minimal power expense.

![Figure 7. Voltage profiles before and after voltage regulation with different objectives](image)

In the second single snapshot simulation, the proposed method, centralized optimization, ADMM and centralized optimization with fairness are simulated with Intel(R) Core(TM) i7-7700K CPU. The voltage is regulated within the limits after three iterations using proposed method with 2.02 seconds. The ADMM can get a relatively good result after 100 iterations which need 45 minutes. The required regulation power from agents is shown in Figure 8. The required active power for each agent has a big difference between those three methods. The centralized fairness method engages all agents equally and uses the most power. The centralized method and ADMM result in the minimal required power, however compared to the proposed method, more power is needed to perform regulation (86.3 and 84.8 vs 58.9) and more agents are required (10 agents vs 8 agents). To appreciate the performance of the proposed method, the regulated voltage profiles between those methods are shown in Figure 9. We can see that the voltage can be regulated in a better way through the proposed method. It is because the voltage limitation constraint in our method is the precondition to quit an iteration, while in other methods, the voltage limitation constraint is handled as the penalty item in the objective function. The final results for voltage regulation performance depends on the setting of the penalty parameter.

![Figure 8. Comparison results between different objectives](image)

### Table I

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Power usage for voltage regulation[kW]</th>
<th>Power losses[kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min contribution power</td>
<td>8.71</td>
<td>82.39</td>
</tr>
<tr>
<td>Min voltage deviation</td>
<td>7.80</td>
<td>180.00</td>
</tr>
<tr>
<td>Min power flow losses</td>
<td>7.99</td>
<td>180.00</td>
</tr>
</tbody>
</table>

### B. Cases and results

The simulation is performed in three parts. Firstly, in case 1 several single snapshot simulations are performed to evaluate the performance of minimizing the usage of power for voltage regulation and to analyse the effect of the proposed method at one time step. Then, in case 2 a time-series simulation in a typical day is performed. Finally, in case 3 a simulation is performed to analyze the robustness of the proposed method to a single node failure.

1) **Single snapshot simulation:** The data of power consumption at one time step in the test grid is set to 20kW and 0.19kVar of all nodes, and the voltage limit is 0.95pu-1.05pu [29]. In the first simulation, the available power for voltage regulation at each agent is within [-10kW, 10kW] and [-10kVar, 10kVar]. We make a comparison between three different objectives. These are: minimizing contributing power for voltage regulation as shown in (Equation 16), minimizing the power flow losses and minimizing voltage deviation. The mathematical formulation of the last two objectives is given in [30]. The total power usage for voltage regulation and power flow losses among these objectives are shown in Table I and the voltage profiles are shown in Figure 7. From Table I we can easily see that the power flow losses in the network do not show a considerable difference between the

2) **Time-series simulation:** To better observe the voltage regulation efficiency of the proposed method, a time-series simulation is performed. The net power consumption at each node follows Figure 6. The available reactive power and active
power for voltage regulation at each agent can vary over the day. However, in the time-series simulation, the available power for voltage regulation at each agent is assumed to be within \([-15\text{ kW}, 15\text{ kW}]\) at all times. The voltage limit is \(0.95\text{pu} - 1.05\text{pu}\) and the time resolution is 15 minutes. The voltage profile at several nodes before voltage regulation is shown in Figure 10. The red dashed lines are the voltage limits. It is easy to notice that the voltage drop problem happens in the morning and evening (7:00-8:00; 18:00-23:00), and the voltage rise problem happens at noon (12:00-16:00).

When the proposed method is applied to regulate the voltage, the voltage profile after the regulation is shown in Figure 11 and the required active power of some agents is shown in Figure 12. It can be observed that the voltage can be regulated within the limits. In this simulation, once the agent indicates their power flexibility for voltage regulation, they will be treated equally. The preference and constraints of agents are expressed through their available capacity for regulation. Within these limits, the effective agents are naturally firstly chosen to regulate the voltage. It is quite possible that the most effective agents will be chosen to provide voltage regulation often and for longer periods, while the ineffective agents will be utilized sporadically. As an example of higher agent utilization, see R1 in Figure 12. Although the agents should take into account their own preference and capabilities for continuous engagement when they report their flexibility for the next time instance (next 15 minutes in this paper), we still propose a weighting factor \(\rho\) to reduce the possibility that a misjudgment will have a high impact on the overall regulation. On one side, the weight factor can make all agents take on voltage regulation tasks more equally, and on the other side, it can avoid assets discomfort caused by the agent taking on the voltage regulation task for long duration. The weight factor is used to find the agent set \(NC_i\) in Figure 3. In this simulation, the \(NC_i, t\) is not the union of agent \(i\) and its nearest neighbors with voltage regulation ability, but defined as the union of agent \(i\) and the neighbors with lowest value of Equation 36 who have voltage regulation ability. The formula for selecting agents using the weighting factor is:

\[
\rho \frac{d_{j,t}}{\sum_{j=1}^{n-1} d_{j,t}} + (1 - \rho) \frac{u_{j,t}}{\sum_{j=1}^{n-1} u_{j,t}},
\]

where \(d_{j,t}\) is the distance of agent \(j\) and the agent with severe voltage problem \(i\) at time \(t\), and \(u_{j,t}\) is the total utilization time of agent \(j\) before time \(t\). The value of weight factor is between zero and one, and it is used to indicate the operator’s willingness to place greater value on effectiveness or fairness when choosing agents to regulate the voltage. The higher the weight factor, the more effectiveness is valued. In Figure 11 and Figure 12, the value of weight factor is 1. Keeping everything else the same, the simulation results with weight factor 0 are shown in Figure 13. By comparing these Figures, it can be seen that the agents are more equally chosen when weight factor is 0. The power providing time period during one day of each agent is shown in Table II. This table shows that the bigger the weight the more fairness in terms of total regulation hours is there. The required power at agent R17 with different weight factor is shown in Figure 14. We see that a lower weight factor prevents the use of one agent for a long continuous period of time. As a downside, the most effective agent is not always engaged in the voltage regulation, leading to a higher contributing power for the regulation.

3) Robustness to a single agent failure: Since neighboring agents are needed to regulate voltage, the situation that an agent suddenly stops cooperating should be taken into consideration. A study is performed in which one agent suddenly stops providing voltage regulation, without a previous notice, and the robustness of the proposed method is observed. The data of power consumption at one time step in the test grid is shown in Table III. The controllable power at each
agent is ±20kW/kVar. Agent R15 is randomly chosen to refuse to provide voltage regulation service. In order to avoid an endless loop and ensure convergence, we set the maximum number of iterations. In this simulation the iteration limitations are set at 20, 50 and 100. The comparison of the results between the proposed method and the centralized method are shown in Figure 15 and Figure 16. It can be seen that the operational agents will provide more power to regulate the voltage, and the voltage can be gradually regulated within the limits. Although 20 iterations is enough to get a good result, the figures also show that the bigger the iteration number, the better voltage regulation. The centralized method reduces the voltage violation from 3.31% to 2.23%, and the proposed method reduces the voltage violation to 0.83%. When compared with the centralized method, the proposed method can make a voltage regulation improvement of up to 122% over a 100 iterations. In conclusion, when compared to the centralized method, the proposed method can gradually reduce the impact of the unexpected event leading to a better overall result.

TABLE III
THE LOAD DATA

<table>
<thead>
<tr>
<th>Load power</th>
<th>R1</th>
<th>R11</th>
<th>R15</th>
<th>R16</th>
<th>R17</th>
<th>R18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active [kW]</td>
<td>190.00</td>
<td>14.25</td>
<td>49.40</td>
<td>52.25</td>
<td>33.25</td>
<td>44.65</td>
</tr>
<tr>
<td>Reactive [kVar]</td>
<td>62.44</td>
<td>4.68</td>
<td>16.24</td>
<td>17.17</td>
<td>10.93</td>
<td>14.68</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper, a new distributed cooperation method for voltage regulation is proposed. Compared to the centralized

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Fig. 11. Voltage profile after voltage regulation by proposed method

Fig. 12. Required power to regulate the voltage by applying proposed method

Fig. 13. Required power with weight factor 0

Fig. 14. Required power of agent R17 with three different weight factors

Fig. 15. Voltage profile after regulation with a single agent failure
control, the proposed method does not have a central controller which is limited by a single point of failure, high communication requirement, and computation burden. Firstly, the proposed distributed cooperation method is naturally reliable and suitable for the complex distribution networks. Secondly, it uses fewer agents to regulate the voltage within the limits. Compared to other distributed methods, the proposed method has a faster response time to the voltage problem. It directly applies the calculated injections in each iteration rather than waiting for convergence before implementation. Considering unexpected errors, the proposed method can automatically regulate the voltage within the limits while relying on cooperation among neighbors. However, in this paper we did not consider the time-series characteristics of the voltage regulation assets nor their internal states. A special case in which one agent need to regulate the voltage rise and the voltage drop problem at the same time is not investigated in this study. In this paper, we simply result to the average value of power if one agent receives different voltage regulation instructions. Those two aspects will be explored further in the future research.

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