The competition and equilibrium in power markets under decarbonization and decentralization: A review

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ABSTRACT

Equilibrium analysis has been widely studied as an effective tool to model gaming interactions and predict market results. However, as competition modes are fundamentally changed by the decarbonization and decentralization of power systems, analysis techniques must evolve. This article comprehensively reviews recent developments in modelling methods, practical settings and solution techniques in equilibrium analysis. Firstly, we review equilibrium in the evolving wholesale power markets which feature new entrants, novel trading products and multi-stage clearing. Secondly, the competition modes in the emerging distribution market and distributed resource aggregation are reviewed, and we compare peer-to-peer clearing, cooperative games and Stackelberg games. Furthermore, we summarize the methods to treat various information acquisition degrees, risk preferences and rationalities of market participants. To deal with increasingly complex market settings, this review also covers refined analytical techniques and agent-based models used to compute the equilibrium. Finally, based on this review, this paper summarizes key issues in the gaming and equilibrium analysis in power markets under decarbonization and decentralization.

KEYWORDS

Power markets, high renewable penetration, equilibrium analysis, modelling methods, solution techniques, recent developments.

1   Introduction

Since the 1990s, several countries around the world have promoted deregulation of the electricity industry[1,2]. In these markets, participants make decisions, compete with each other and game the rules for their own interests. The electricity resources are allocated by the market mechanism, and the prices are formulated to encourage competitive behavior.

Unlike in a vertically integrated monopoly mode, the market clearing results are not controllable by a single operator and are determined by competition instead. To this end, market equilibrium analysis emerges as an effective tool to study gaming interactions and predict potential market outcomes. From the perspective of a market monitor, this type of analysis helps detect potential market malfunctions and evaluate market power[3]. From the perspective of market participants, such analysis helps assess the market situation and make favorable bidding decisions[4].

Equilibrium analysis finds its origin in Nash equilibrium theory and depicts a stable market situation as such that no one has an inventive to deviate from it[5]. Depending on the bidding format, traditional equilibrium analysis models can be categorized into Cournot[6] and supply function equilibrium models[7]. To better capture the complicated clearing rule, a bi-level model is proposed, which models participants as the leaders and the market clearing as the follower. This bi-level model can be recast as the mathematical programming with equilibrium constraints (MPECs)[8] or equilibrium programmings with equilibrium constraints (EPECs)[9] and solved using optimization techniques.

There have been several articles reviewing the market equilibrium analysis, both on the traditional models[10,11] or the optimization-based MPEC and EPEC models[12,13]. These reviews mainly focus on competition in wholesale energy markets dominated by thermal generators. However, due to the trend towards decarbonization and decentralized generation in power systems, competition in power markets is also undergoing fundamental changes, which leaves traditional equilibrium analysis insufficient.

On the one hand, wholesale markets are embracing new participants and introducing new trading products to accommodate the increasing penetration of renewables. Renewables, storage and aggregators gradually enter a market traditionally dominated by thermal power plants, and novel trading products are also introduced to increase flexibility, manage, risks and reward renewable generation. These changing market structures and mechanisms alter the gaming behaviors and market equilibrium.

On the other hand, competition in increasingly decentralized power systems is gradually extended to emerging markets, such as distribution markets and the aggregation of distributed energy resources (DERs). As shown in Figure 1, the emerging markets are usually organized in the distribution network, and the aggregators can act as an intermediary between the wholesale market and the DERs. Distribution markets may be organized in novel modes such as peer-to-peer (P2P) trading, where gaming is conducted in a distributed and repetitive way. Unlike the non-cooperative Nash game in the wholesale auction, the aggregation of DERs can be modelled as a cooperative or Stackelberg game, and the analyzing tools are heterogeneous.

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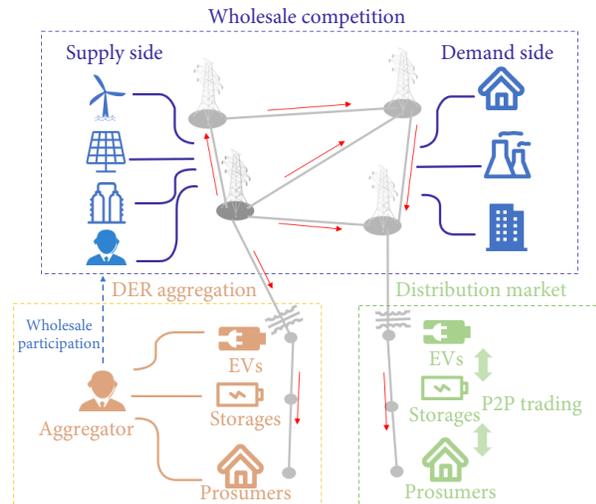
Fig. 1   The various competition modes in the market.

This variety of competition modes makes the traditional "perfect information and rationality" assumption impractical. Heterogeneous information acquisition degrees, risk preferences and rationality degrees of market participants must be considered. Thanks to the increasing availability of market information, data-driven methods can be used to reveal these characteristics.

The emergence of novel competition modes has also promoted the development of equilibrium solution algorithms. In previous research where the settings are relatively simple, the equilibrium model can be solved by diagonalization or Karush–Kuhn–Tucker (KKT) methods. With the growingly complex market settings, such models may be intractable, and customized analytical techniques are needed to recast them. Due to its compatibility with the complex market setting and participants' preferences, agent-based models are further explored to enhance learning and convergence performance.

To sum up, the changing electricity market competition and equilibrium have aroused the attention of many researchers, and there is an urgent need to summarize these works. This article reviews recent studies on equilibrium analysis, summarizes developing trends and outlines future challenges.

The rest of the paper is organized as follows. Sections 2, 3 and 4 review the gaming analysis in the changing wholesale market, the emerging distribution market and DER aggregation, respectively. Research on information, risk attitude and rationality settings are reviewed in Section 5. Section 6 reviews the new analytical techniques and agent-based models to compute the equilibrium. Section 7 points out the possible challenges in equilibrium analysis. Section 8 concludes the paper.

2   Gaming in wholesale markets with increasing renewables

Hosting a considerable share of trading volumes, the wholesale market is nonnegligible in the gaming analysis. Guided by the carbon neutrality target, the traditional thermal-dominated electricity market is embracing decarbonization. On the one hand, new entities such as renewables, storage and aggregators are taking increasingly important positions. On the other hand, novel trading categories have been introduced in several markets to help participants hedge risks, recover investment costs, trade flexibility, or reward green energy. The changes in participant structure and

mechanism design will synergistically change the competition mode and market equilibrium.

It is notable that some literature assumes a single participant as strategic and other as non-strategic. In this case, the EPEC which characterizes a multi-leader-one-follower game can be simplified to the MPEC which characterizes a single-leader-one-follower game. The single strategic participants try to maximize their profit while the market operator tries to maximize social welfare, and the Stackelberg equilibrium can be derived. Although simplifications are made, these researches also take an important position in the market competition and equilibrium analysis. On the one hand, the participants with large market share and strategic behaviors are limited, and most small participants act as price-takers and behave honestly. On the other hand, the MPEC is the theoretical foundation of EPEC. After properly characterizing one strategic participant by MPEC, the EPEC can be obtained by jointly establishing several MPECs.

2.1   Entry of new gaming participants

Under the decarbonization target, renewables are becoming increasingly important participants in the market. Meanwhile, to tackle the uncertainty of renewables, several flexibility providers, such as demand responses and storage, have emerged alongside renewables. The DERs can also take part in the markets after aggregation in the format of aggregators, virtual power plants (VPPs) or retail companies. These market participants have different physical constraints and operation feasible regions, and in consequence, the modelling of their decision problem differs.

For renewable participation, bidding behaviors are closely related to stochasticity. Tsimopoulos and Georgiadis use the EPEC model to study the stochastic equilibria in markets with high renewable penetration[14]. To reduce imbalance costs, He et al. propose cooperative participation of wind producers and energy storage and derive the bidding behaviors of the coalition in the energy and regulation market[15]. The strategic behaviors of wind producers who can withhold capacities on the excuse of prediction uncertainty have been studied[16,17].

Accompanying renewables, many participants providing flexibility and reliability are entering the market. When deriving the bidding behaviors of cascaded hydropower stations, the coupled constraint between upstream generation and downstream flow input is considered[18]. Pan et al. study the bidding strategies of power to hydrogen and methane (P2HM) plants on electricity, gas and hydrogen markets. Specifically, the energy transfer equations are considered[19]. The profit that storage can earn by providing reserves and balancing services is measured[20].

The aggregation of DERs can participate by agents. A Nash–Cournot equilibrium model is proposed to analyze the impacts of VPPs on market operation[21]. Considering uncertainties, including electric vehicle (EV) fleet flows and hourly load profiles, a stochastic equilibrium involving EV aggregators is analyzed[22]. Liu et al. analyze the impact of wind aggregator bidding on the joint equilibrium of energy and ancillary services markets[23]. Carrion et al. use the MPEC model to study the strategic behaviors of retail companies and consider their pricing package designation to consumers when making bids[24].

By modelling the decision problems of new market entrants, their optimal bidding strategies can be derived, and several kinds of novel gaming behaviors can be detected. By calculating the formulated equilibrium, these studies help us to evaluate the market impacts of the entry of new participants.

2.2　Emergence of the multi-stage and multi-product market

With the improvement of market mechanisms and the introduction of new trading categories, the competition in the single energy market is gradually evolving into a complicated multi-market competition. The equilibrium analysis of multiple markets can be quite challenging. Various markets have different trading rules and market structures. In the absence of generalized modelling methods, they must be modelled on a case-by-case basis. Furthermore, the feasible regions for bidding in different markets are mutually constrained and earned revenues constitute a trade-off, so the multi-market competition is a complex joint gaming problem.

2.2.1　Multi-stage market competition

The introduction of the variable renewables pushes up price volatility, prompting participants to use more risk hedging tools and participate in multi-stage markets. Kardakos et al. study the day-ahead and real-time two-stage market equilibrium with a strategic virtual power plant that tries to minimize its imbalance costs[25]. Fang et al. study how deviation penalties can regulate wind generators' behaviors by analyzing the equilibrium[26]. The joint equilibrium model of spot markets and long-term markets is established, and the impact of long-term contracts on mitigating market power is analyzed[27]. The cross-market arbitrage of thermals by using long-term contracts is analyzed, and the corresponding impact on spot market equilibrium prices is evaluated[28].

Apart from the competition in daily market operation, the competition has been extended to capacity remuneration mechanisms due to the lowered energy prices. The two-stage equilibria, including investment and bidding, is analyzed where the strategic behaviors of wind producers are considered[29]. Grimm et al. analyze the joint investment equilibria of transmission and generation, and the impact of pricing zone design and renewable policy is considered[30]. Chen et al. analyze the investment decision of gas-fired units. By modelling the operation of gas and electricity markets, the expected revenue can be back fed to the investment stage[31].

2.2.2　Multi-product market competition

To ensure the reliable and flexible operation of the system, several new products, such as flexibility and ramping products, have been introduced in recent years. Meanwhile, products such as reserve, regulation and capacity are becoming increasingly important revenue sources for market participants. There have been many studies analyzing the equilibrium of markets with new trading products. The EPEC model is used to study the clearing of the energy and reserve markets[32]. The joint clearing mode and sequential clearing mode are examined and their formed equilibria are compared. Zou et al. use the Cournot model to analyze the joint gaming behaviors in energy and ancillary markets, and they find that energy prices decrease while ancillary services prices increase with higher renewable penetration[33]. The flexible ramping product is introduced in California independent system operators (CAISO) to purchase ramping capabilities, and its market equilibrium is examined by a multi-period Nash–Cournot equilibrium model[34].

Additionally, to incentivize the development of green energy, several green financial products are launched to reward their environmental friendliness. Helgesen and Tomasgard study the green license and analyze its impact on the operation of the electricity market by calculating the joint equilibrium[35]. The impact of carbon policy on market equilibrium is studied, and the carbon tax is found to be the most cost-efficient tool to reduce emissions[36]. Based on California data, Hu et al. use the EPEC model to quantify the impact of carbon prices on electricity market equilibrium and compare the competitiveness of generators with different fuel types[37].

In summary, more refined equilibrium models have been established to analyze the multi-stage and multi-product wholesale market. The references are compared in Table 1. Cross-market arbitrage, risk management and investment behaviors can be analyzed and predicted. The impact of new trading products on market prices, clearing quantities and social welfare can be determined.

3　Gaming in the emerging distribution markets

Decentralization is an important trend of the future market. Local

Table 1　Comparison of reviewed literature on the wholesale market

Reviewed literature	Participant	Trading stage	Trading product
[14, 17, 26]	Renewables	Day-ahead, balancing	Energy, deviation penalty
[15]	Wind and storage	Real-time	Energy, regulation
[16, 29]	Wind	Investment, day-ahead, balancing	Capacity investment, energy
[18]	Hydropower stations	Day-ahead, real-time	Energy
[19]	P2HM plant	Day-ahead	Electricity, gas, hydrogen
[20]	Storage	Day-ahead, real-time	Reserve, balancing services
[21–23]	VPP or aggregators	Day-ahead	Energy, reserve, regulation
[24]	Retail companies	Day-ahead	Energy (wholesale), customized price design
[25]	VPP	Day-ahead, real-time	Energy, deviation penalty
[27, 28]	Thermals	Long-term, spot market	Energy
[30]	Investor	Investment, spot market	Transmission and generation investment, energy
[31]	Gas units	Investment, spot market	Capacity investment, energy, gas
[32]	Wind	Investment, day-ahead	Capacity investment, energy, reserve, subsidy
[33]	All kinds	Day-ahead	Energy, reserve, regulation
[34]	All kinds	Day-ahead	Energy, flexible ramping
[35–37]	All kinds	Day-ahead	Energy, green products (certificates, carbon tax, carbon prices)

power balance may be realized at the distribution grid level, and entities form a local market to trade energy and flexibility. With the penetration of DERs and the implementation of vehicle-to-grid (V2G) technology, novel market participants named pro-sumers that can both consume and produce electricity have emerged[38]. The distribution market bears fundamental changes compared with the traditional wholesale market. Firstly, the trading mode will change due to the different operating nature of the dis-tribution grid. Secondly, novel trading products need to be intro-duced to tackle the operating challenges in the near-island small-scale grid with high renewable penetration. The new trading mode and trading products will, consequently, change the landscape of gaming behaviors in the distribution market.

3.1   The new trading modes

The possible trading modes of distribution markets under heated discussion include distributional locational marginal price (DLMP) and P2P energy trading. There have been several studies elaborating on gaming behavior analysis in the corresponding market.

3.1.1   Competition under the DLMP mechanism

DLMP resembles the concept of the pool-based wholesale market, and it can also be derived as a byproduct of the clearing[39]. The distribution market clearing is based on alternative-current optimal power flow (ACOPF), which considers voltage constraints and power losses. Participants can game on the new constraints, and how the demand side can profit by providing reactive power, voltage support and congestion mitigation in the distribution market has been explored[40,41]. The strategic behaviors of multiple participants are considered by EPEC, and the formulated equilib-rium is calculated[42].

The participants in the DLMP mechanism can be diversified. The aggregators can act as a proxy for distributed prosumers and respond to the DLMP released by the distribution system operator (DSO)[43]. The EV aggregator[44], energy storage[45] and microgrids[46] can also be the participants in the distribution market, and the corresponding equilibria are analyzed.

3.1.2   Competition under P2P distributed trading

In the distribution market, the boundaries between buyers and sellers are blurred due to the emergence of prosumers. The exchange could be bidirectional, unlike the unidirectional exchange in the wholesale market. With the help of distributed optimization techniques (e.g., ADMM), many studies have turned to distributed trading from the original pool-based market. In comparison with the DLMP mechanism, P2P distributed trading has three major advantages: (1) privacy reserving; (2) user-centric, i.e., trading pairs are formed during market clearing in accordance with user preferences[47]; (3) more flexible structures, i.e., massive participants could asynchronously trade together[48]. Since most distributed optimization techniques rely on iterations between market participants, the clearing procedure of P2P trading is more complex than the DLMP mechanism, especially in hierarchical markets.

Since the distributed market mechanism is still a research frontier and there is no industrial-level standard for P2P distributed energy trading, game theory and equilibrium analysis are mainly used for mechanism design. Paudel et al. use an M-leader and N-follower Stackelberg game to model the interaction between sellers and buyers in a prosumer-based community[49]. The buyers are assumed to play an evolutionary game once the sellers announce the price vector after a noncooperative game. Le Cadre et al. propose an optimization model for prosumers to determine their trades, demand, and flexibility activation and characterize the solution of P2P trading as a variational equilibrium[50]. The distributed energy trading and benefit allocation problem is formulated as a general Nash bargaining problem, and ADMM is used to solve the problem [51]. Motivational psychology models are used to design a game-theoretic P2P trading scheme[52]. However, the physical constraints of the distribution grid are not considered in the scheme.

In summary, gaming and equilibrium in distribution markets take new formats. The DLMP mechanism has fewer differences since it inherits the competition mode of the pool-based market as a first-price sealed-bid auction. It is notable that the gaming and competition circumventing reactive power and voltage support becomes more predominant. In contrast, P2P trading changes the competition mode to iterative gaming. The exchange can be con-ducted asynchronously between any pair, and the market results converge to equilibrium after multi-round information exchange. The comparisons of DLMP and P2P mechanisms are shown in Figure 2.
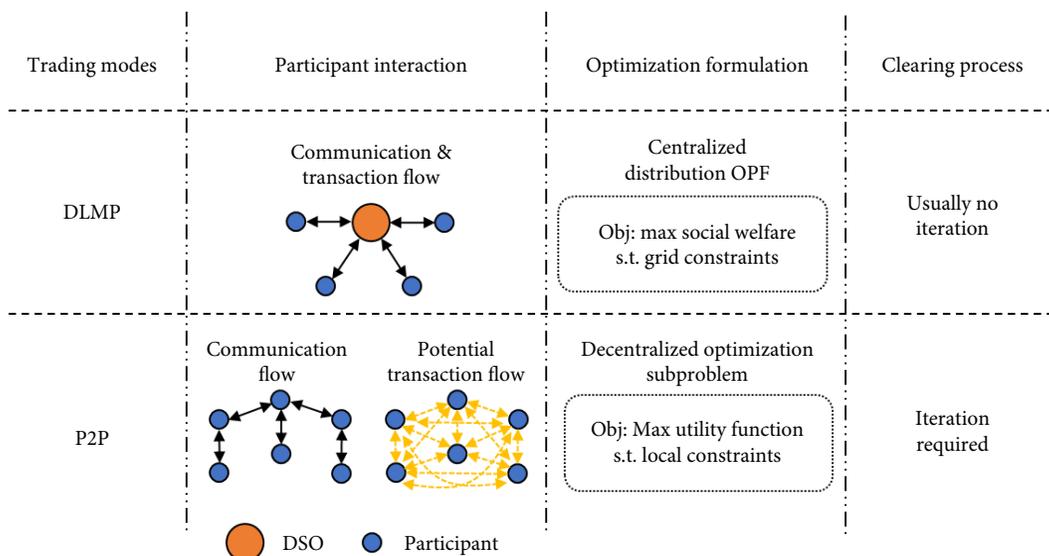

Fig. 2   Comparison between the DLMP and P2P mechanisms (Note: The communication flow of P2P would be similar to that of DLMP in those cases when a DSO is required to coordinate the distributed clearing process).

3.2 Novel trading products

To ensure flexible, reliable and low-carbon operation of the distribution grid, several studies have proposed that the market should introduce novel trading products, including flexibility, reserves and carbon products. Unlike the wholesale markets, the new products in the distribution grid are traded under the P2P or DLMP framework.

Under the DLMP framework, several studies embed new products in the ACOPF clearing model, and the gaming behaviors are analyzed. The values of voltage support and congestion management are reflected in different components of the DLMP[53]. A chance-constrained ACOPF model can formulate the uncertainty price, and its impact on incentivizing imbalance mitigation is presented[54]. Similar work is conducted by a robust optimization method[55]. Lu et al. analyze the impact of incorporating capacity tariffs in DLMPs on participants' behaviors[56]. A carbon-aware DLMP that incorporates emission costs is proposed to enable joint trading of energy and carbon[57].

Under the P2P trading framework, Zhang et al. propose a joint trading mechanism involving energy and flexibility and analyze the market equilibrium[58]. Similar work is conducted, with the reserve requirement determined by chance constraints[59]. The energy storage usage right is treated as a new product and traded in P2P ways[60]. Hierarchical demand response is proposed, which can effectively exploit flexibility from the consumer side[61]. To reflect the intertemporal differences in energy value, Fan et al. decomposed the power curve into two products named energy level and energy shift[62]. The analysis of the equilibrium shows that the energy shift can incentivize peak shaving.

4 Gaming between aggregators and distributed resources

The variability and intermittency of variable renewables exert greater challenges on the wholesale market operation. The wholesale market needs to fully exploit the flexibility from the increasing DER, including prosumers, renewables and EVs. Aggregators, therefore, emerge as an important intermediary between the wholesale market and distributed resources, and their gaming with the distributed resources has aroused great attention. Generally, there are two interaction modes between aggregators and DER: a cooperative game and a Stackelberg game. Recently, there has been literature that incorporates aggregators and DERs in the P2P trading framework, where the aggregators can trade with DERs bilaterally and interact with the main grid[63,64]. These researches provide meaningful explorations, and P2P might be another aggregation mode in the future.

4.1 Analysis of cooperative games

In cooperative games, an aggregator can be regarded as a coalition formed by DERs. For the sake of narration, we do not distinguish the concepts of VPP and aggregator. Based on the aggregated operating characteristics of the DERs, the aggregator gains profits by bidding in the electricity market and then allocates the profits to the DERs according to their contributions. Strictly speaking, there is no conception of equilibrium in cooperative games since the participants do not make independent bidding decisions. What they need to decide is whether to form or leave the coalition, and the profit allocation and core stability concept in the cooperative game theory resembles the equilibrium in the non-cooperative game.

4.1.1 Why form coalitions

In current electricity markets, energy deviations are penalized or traded in the balance (real-time) market. Therefore, due to the uncertainty of generation, the profits of DERs are usually reduced. As a result of the noncorrelation of DER generation fluctuations[65] and further, the integration of flexible resources such as energy storage units[66], the generation uncertainty of the aggregated DERs decreases, and the aggregator gains more total profits than when the DERs operate individually[65–67]. The increased profit, or so-called surplus profit, is why coalitions of DERs can exist.

4.1.2 Profit allocation method and core stability

In the literature using coalitional game theory to study the interaction between DERs and aggregators, most researchers focus on profit allocation (Table 2). Specifically, the goal is to design profit

Table 2 Comparison of the reviewed literature on cooperative games

Reviewed literature	DER type	Market environment	Allocation method
[65]	Wind power	Two-settlement power market	Minimum worst-case dissatisfaction
[66]	Non-dispatchable producer, energy storage, non-dispatchable load	Two-settlement power market	Nucleolus and the Shapley value methods
[67]	Non-dispatchable producer, energy storage, non-dispatchable load	Two-settlement power market, reserve market	The virtual internal transactions
[68]	Non-dispatchable producer, non-dispatchable load, reducible load	Two-settlement power market	The bi-objective optimization framework
[69]	Wind power	Two-settlement power market	Core selection with arbitrary criteria
[70]	Renewables	Day ahead market with penalty prices	The cost causation based framework
[71]	EV, wind power, non-dispatchable load, shiftable load	Day-ahead market, reserve market	The Aumann–Shapley method
[72]	Renewables	Only generation considered	Nucleolus, the Shapley value, and minimum cost-remaining savings
[73]	Renewables, non-dispatchable load, interruptible load	Time of use price and government subsidy	The Shapley value method
[74]	EV	Day-ahead market, regulation market	Trading between the aggregator and EV
[75]	Renewables	Two-settlement power market	The stabilizing contract
[76]	Renewables	Payment from the grid based on the accuracy	Production and prediction-based scoring rule
[77]	Renewables	Two-settlement power market	A specially formulated market

allocating methods with the desired properties of stability and fairness according to the specific market environment and the characteristics of the DERs. The proper allocation of profit and costs determines the coalition stability.

The early work of Baeyen et al. examines the aggregation of wind power plants to maximize the expected profit in a two-settlement energy market, where the realized profits are allocated to the plants minimizing the worst-case dissatisfaction to make the allocation stable[65]. Dabbagh et al. consider an aggregator consisting of photovoltaic power plants, wind power plants, energy storage systems and loads, which bids in a dual pricing market by risk-averse two-stage stochastic programming, and the surplus profit is allocated by nucleolus and Shapley value methods[66].

However, the nucleolus and Shapley value methods can only ensure the stability and fairness of allocation, respectively, rather than achieve the two goals at the same time, and the computational complexity of the methods is very high. Building on previous works, a virtual transaction mechanism within the aggregator is designed to decompose the contribution of DERs and allocate profits accordingly[67], which is more comprehensible and computationally efficient. Aiming at an efficient tradeoff between stability and fairness, a bi-objective optimization-based method is proposed to allocate the cost of an aggregator[68]. Nguyen et al. propose a method to quickly solve the core selection problem in a coalition of wind power plants through a constraint generation algorithm[69]. For computational tractability, cost causation-based[70], Aumann–Shapley value[71], and minimum cost-remaining savings[72] methods for profit (cost) allocation within aggregators have been used to replace the Shapley value method in specific scenarios. In addition to the abovementioned DER types, there are also recent studies on DER coalitions containing interruptible loads[73] and EVs[74].

4.1.3   *Coupled aggregator bidding and profit allocation*

In the above studies, the bidding of an aggregator and the coalition game within it are decoupled, meaning that the optimal bidding strategies of aggregators can be used directly. However, although the expected profits of aggregated DERs are in many cases super-additive[65–67], the realized profits are not necessarily superadditive, which may affect the stability of the alliance[75]. Meanwhile, it is possible that the DERs strategically submit false generation forecasts, so the allocation method should ensure that the best strategy for the DERs is to provide forecasts that are as accurate as possible[76,77]. In other words, without loss of generality, incentive compatibility for DERs needs to be ensured when aggregators design allocation methods, but this is rarely considered in existing studies.

4.2   **Analysis of Stackelberg games**

Another gaming mode between aggregators and DERs is the Stackelberg game, where the aggregators act as leaders and set incentives to the DERs who act as followers. Different from the cooperative mode, in this mode, the DERs selfishly act on behalf of their own interests, and the aggregators should carefully design the incentives based on the estimation of followers' responses. The interaction can be modelled as a Stackelberg game, and the Stackelberg equilibrium can be solved after deriving the optimality conditions of the follower decision problem and recasting the model to a single-level problem. The incentive design and varied scenario analysis have to be based on the Stackelberg equilibrium analysis.

4.2.1   *Varied incentives design*

The incentives may vary, including flat-rate tariffs, time of use (TOU), real-time pricing (RTP), critical peak pricing (CPP) and one-time rewards. Several studies use bi-level models and compare the derived Stackelberg equilibrium. Grimm et al. find that RTP is more favorable for retailers in terms of expected profit and risk management[78]. Ansarin et al. find that TOU and RTP prevail over other methods with respect to efficiency and fairness[79]. Zhang et al. design CPP parameters to guide DER performance to minimize the total operational cost, including energy purchasing cost and imbalance penalties[80]. It is found that the coordination of fixed and real-time dynamic tariffs can better incentivize peak shaving[81]. Althaher et al. establish an online algorithm and design a price threshold in the RTP to prevent homogeneous adjustment and the incurred new peak hours[82].

4.2.2   *Consideration of practical economic and physical factors*

Specifically, several practical factors need to be considered when designing pricing strategies.

Typical characteristics of DERs include response uncertainties, consumer psychology and dispatch merit order. In the work of Mocanu et al., the users in the smart building are guided by one-time incentives, and a deep reinforcement learning algorithm is used to properly deal with the uncertain responses[83]. Consumer psychology is considered to better capture consumers' behavior patterns during contract signing[84]. When providing balancing services, the dispatch merit order is formulated based on the biddings of DERs[85].

Technical aggregators such as microgrids and DSOs also need to consider physical constraints. The voltage constraint is considered when determining pricing strategies[86]. The three-phase balance constraint in the distribution grid is considered in the aggregation for the DSO[87]. Zheng et al. consider the diversified operation characteristics and responses from multi-energy carriers[88].

4.2.3   *Simultaneous gaming for aggregators in the wholesale and aggregation market*

As the intermediary agent, the aggregators simultaneously game with other wholesale market participants and the DER, and the joint optimal strategies in retail pricing and wholesale participation are worth noting. The strategic behaviors of the DSOs in the wholesale market that aggregate DERs are studied by an MPEC model[42]. The robust model, conditional value-at-risk method and fuzzy optimization can be used to help retailers manage imbalance costs arising from consumer response[89–91]. Sarker et al. propose a more refined two-stage pricing framework for the retailer to better manage imbalance risks, which includes a day-ahead prescheduling stage and a real-time rescheduling stage[92].

4.3   **Equilibrium comparison between the two modes**

As the two most common gaming modes between aggregators and DERs, the cooperative game and the Stackelberg game have strengths and weaknesses in terms of incentives and efficiency.

4.3.1   *Incentives*

The profit or cost allocation is performed ex-post in the cooperative game, and guaranteeing core stability and fairness remains a challenge. Improper allocation might undermine DERs' incentives to participate in the coalition.

The Stackelberg game leaves room for DER to freely decide their best responses based on the released incentives. They will be incentivized to remain in the game and make commitments under advantageous incentives.

4.3.2   *Efficiency*

The cooperative game can more effectively utilize all the resources

since the control right is ultimately relegated to the aggregator. The full potential of profit can be realized for the coalition, and from the system perspective, distributed flexibility can be effectively utilized.

In the Stackelberg mode, the effective design of retail pricing highly depends on the accurate estimation of DER preferences and satisfaction. The ideally optimal result may not be realized in practical conditions due to information asymmetry.

5 Individual bidding characteristics in competition modelling

According to the assumptions of traditional studies on market competition and equilibrium, the modelled participants can usually be seen as "perfect-rational-man", with features such as complete-information or perfect-rationality. However, the researchers gradually find that these assumptions are too strong, distorting the individual bidding characteristics of the participants in the actual market operation, which leads to inaccurate participant modelling and biased market competition simulation. Thus, it is necessary to consider individual bidding characteristics during participant modelling and market competition analysis. In recent years, studies on the individual bidding characteristics of participants during their decision-making process have attracted increasing attention.

At present, the modelled individual bidding characteristics can be roughly divided into two categories. The first category is the information characteristic, representing the various information acquisition degrees of different participants, which determines the external boundary of the individual decision-making. The second category is the decision characteristic, which represents the various inner features that appear during the individual decision-making process and influence the final bidding outcomes, such as risk-preference and rationality degree.

5.1 Settings of information characteristics

The information characteristics are mainly reflected in different information acquisition degrees, which are divided into three types: limited market environmental information, limited market boundary information and limited rival strategy information.

In the first type, the limited information characteristics of participants in market environments, such as power system topologies and transmission congestion lines, have been fully used in many studies to simplify power system operation models[15,93,94].

In the second type, the limited information characteristics of participants on market boundaries, such as renewable generation and system loads, have been fully studied. With increasing penetrations of variable renewable energy in power systems, the uncertainty in power system operation significantly increases, which makes this characteristic more important in participant modelling. For example, the high uncertainty of wind generation is formulated and integrated into a Cournot game to analyze its influences on an energy-only power market[95]. A stochastic agent-based model is developed to analyze the bidding behaviors of renewables with uncertain power outputs[96].

In the third type, the limited information characteristics of participants on rival strategies, such as individual bidding strategies or market competition situations, have been formulated in many studies. Since it is impossible to directly know how the rivals will bid for the DA market, many studies use scenario-based[97] or response-function-based[46] methods to model the rival bidding uncertainty.

5.2 Settings of decision characteristics

Apart from the information characteristics, the various decision characteristics also have a huge influence on the simulative strategic bidding behaviors of participants. In detail, the decision characteristics are defined as the factors that drive participants to deviate their decisions from those made under the perfect-rational-man hypothesis. Two decision characteristics are typically used in market competition analysis: risk preference and bounded rationality.

5.2.1 Risk preference

In actual power market situations, many participants not only attach importance to profit expectations but also pay close attention to the financial risks of their bidding decisions. This phenomenon is more apparent in market participants with higher uncertainties, such as renewable energy generators, load aggregators and energy storage systems.

Risk preferences can usually be divided into three degrees: risk aversion, risk neutrality and risk seeking. Many participants with higher uncertainty are more likely to be risk averse. For example, distribution companies that buy energy from the wholesale market and sell it in the retail market face uncertainty from renewable energy and demands. They are usually assumed to be risk averse[98]. A risk-averse optimal bidding strategy is proposed for EV and energy storage aggregators to participate in day-ahead frequency regulation markets[99]. The risk-averse features of various generators are formulated, and the influences of risk preferences are analyzed based on an EPEC model[100]. This proves that different risk-averse levels and market ownership structures will result in different market equilibria, demonstrating the importance of formulating risk preferences in market equilibrium analysis. Recently, Moret et al. discuss the heterogeneous risk preferences in decentralized power markets and analyze how they change the market equilibrium and payments[101].

To control risks, many methods are utilized and integrated into decision-making process modelling. For example, an optimal bidding strategy for a strategic wind power producer is proposed by integrating conditional value at risk (CVaR) to control the revenue risks caused by wind generation uncertainty[102]. A robust bidding strategy for a hybrid energy generation company (GENCO) constituted by energy generation and retailing business is proposed[103], which is modelled as a max-min bilevel MPEC, while the risks of rivals' uncertain bidding strategies are handled by robust optimization. An information gap decision theory (IGDT) approach is also developed to help microgrids make risk-constrained optimal bidding strategies[104].

5.2.2 Bounded rationality

Compared with the concept of risk preferences, the bounded rationality concept in power markets is a newcomer. Bounded rational theory is put forward and developed by Daniel Kahneman, Amos Tversky and Richard Thaler, who are all Nobel-prize winners. This theory now has a well-established body of work in the fields of economics and psychology. The core idea of bounded rationality used in the power market area is that many participants often seek a satisfactory rather than optimal solution. Several methods from bounded rational theory have been introduced into the power market area, including prospect theory[105] (in the form of framing effects[106] or weighting effects[107]) and fairness standards[108].

Since the bounded rationality of retail participants such as residential houses is more apparent, the bounded rational formulation is first introduced into studies in the field of retail power markets.

For example, the bounded rational behaviors of an active consumer under variable electricity pricing in retail markets are formulated based on prospect theory and a Stackelberg game. The impacts of irrationality on customers and aggregators are both analyzed. Furthermore, bounded rationality has been introduced into many other aspects, such as demand-side management[109], EV charging[110], and virtual power plant control[111].

With increasing evidence proving that some participants in wholesale markets also have bounded rational features, some studies also try to integrate bounded rational formulation into the bidding decision-making process and equilibrium analysis. First, the bounded rationality of GENCOs in day-ahead power markets with bilateral contracts is formulated based on the weighting effects of prospect theory[112]. The GENCOs are further modelled in a game, and the effects of bounded rationality on the market equilibrium have been analyzed. Then, the bounded rationality of duopoly power providers is formulated, and the interaction process is modelled as an evolutionary game[113]. The Nash equilibrium of the market and how information asymmetry affects the results are also discussed. Furthermore, a more detailed model of GENCOs with bounded rationality is developed, where both prospect theory in the form of framing effects and fairness constraints on profit-seeking are formulated[114].

5.3   Methods to reveal individual characteristics

Although the individual bidding characteristics have been increasingly utilized in the modelling of the bidding decision-making process, their exact parameters are usually unknown and determined by expert experiences, which would inevitably cause deviations in individual bidding modelling. To overcome this problem, many studies have proposed methods to reveal the individual characteristics hidden below numerous market data. These methods can be roughly divided into two types: The first type is to enhance the information acquisition ability to overcome the challenges caused by information asymmetry; The second type is to identify and quantify the individual decision features that are naturally inapparent, such as risk preferences or rationality degrees. Figure 3 shows how historical data can empower equilibrium analysis.

5.3.1   Information acquisition enhancement

Many methods have been proposed to solve the problems caused by information asymmetry, such as the limited information of power system topology, rivals' operation costs or bidding strategies, and future power market competition situations.

Many works have been performed to recover the grid topology based on publicly available information. For example, Kekatos et al. use sparse matrix decompositions and congestion information to reveal the power transfer distribution factor (PTDF), a topology-related matrix of the power market[115]. Birge et al. then apply the inverse optimization method to the shadow prices of constraints and LMP data to estimate the PTDF[116]. Recently, Zheng et al. propose an unsupervised approach to analyze the fundamental distribution of the congestion part LMPs in high-dimensional Euclidean spaces[117]. The subspace attributes of an LMP vector under various congestion status of all the transmission lines are also discussed.

Many studies have been proposed to speculate on rivals' bidding strategies. For example, an online learning algorithm based on sparse Bayesian learning for GENCOs is developed and can be used for each GENCO to build a private probabilistic model based on the dynamic Bayesian network to infer rivals' optimal bidding behaviors in the future[118]. The GENCOs are further modelled as agents, and the market equilibrium is analyzed. Later, an inverse optimization approach is proposed to estimate rivals' variable cost functions[119]. Many historical market data, such as market clearing prices and individual cleared capacities, have been used as model inputs.

To unveil future power market competition situations, many studies have been proposed to infer or predict aggregate supply curves (ASCs) or residual demand curves (RDCs). For example, a Bayesian inference approach is developed based on Markov chain Monte Carlo and sequential Monte Carlo methods, which can be used to infer ASC in day-ahead power markets[120]. Later, a feed-forward neural network method is developed to predict ASCs. However, the method is only tested on markets with no more than 12 participants and artificial data[121]. Recently, a novel forecasting model is proposed to help market participants and operators predict ASCs based on principal component analysis (PCA) and the long-short-term memory (LSTM) model[122]. It should be noted that this paper is the first to utilize actual historical market data to predict ASC, and a detailed ASC data processing method is also introduced.

5.3.2   Decision feature identification

Apart from identifying observable parameters such as system topology or operation cost, many studies try to identify the nonobservable parameters underlying the participants' bidding behaviors, such as risk preferences or bidding preferences. Due to
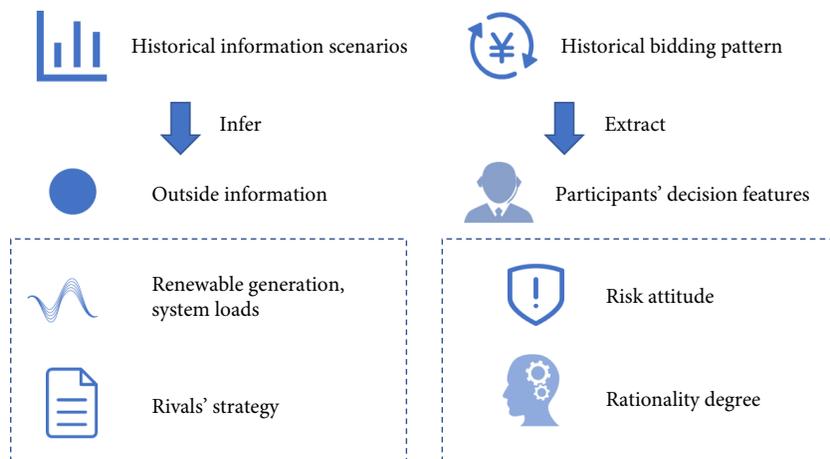
Fig. 3   Use of historical data to empower equilibrium models.

the development of data-driven analysis methods, this kind of research has gradually developed in recent years.

First, a data-driven risk preference analysis method for generators to participate in DA energy markets is developed based on inverse reinforcement learning algorithms. The algorithm is tested on actual market data of the Australian Energy Market Operator (AEMO)[123]. Recently, some researchers have found ways to reveal the individual bidding preferences of participants from market data. A deep inverse reinforcement learning approach is applied to generators in energy markets to identify their bidding objective functions[124]. The extracted reward functions have been modelled as a feed-forward neural network to demonstrate its high-dimensional nonlinear mapping from market status, such as clearing prices and capacities, to its perceptive rewards. Then, an improved inverse reinforcement learning method is proposed to reveal the bidding preferences of energy storage systems in multiple markets, including two energy (up and down) and six frequency control ancillary service (FCAS) markets[125].

6 Development in equilibrium solution algorithms

The changes in competition modelling have also promoted the development of solution algorithms. In previous research where the settings are relatively simple, the equilibrium model can be solved by basic analytical techniques. However, with the increasingly complicated market setting, the traditional analytical model may be intractable. To this end, several studies have provided refined techniques within the scope of analytical solutions. On the other hand, the agent-based model is further explored to adapt to the increasingly complicated market environment.

6.1   Refined analytical techniques

The recent research focuses on equilibrium analysis in the more practical MPEC and EPEC models. Several techniques have been explored to handle nonlinearities in bi-level problems, and equilibrium verification and choice are the subjects of heated discussions.

6.1.1   The refined techniques in solving MPEC

The complicated market setting will pose challenges to the analytical algorithm of the bi-level equilibrium model. Traditionally, the strong duality theorem or KKT condition is used to derive the optimality condition of the lower-level linear programming problem, and the bi-level problem can be transformed into MPEC[126]. However, the introduction of integers and nonlinearities will pose challenges.

The modelling of unit commitments or grid reconfiguration will introduce integers in the lower-level clearing, but the KKT conditions cannot be derived for integer programming. The divide-and-conquer strategy is proposed where iterative methods and cut-adding are used. The feasibility cut and optimality cut can be obtained when solving the lower-level problem and added into the upper-level problem sequentially to approximate the lower-level optimality conditions[127, 128]. The convergence within finite steps has been proven[129], and a gap tolerance can be designed to save computational time[130].

The consideration of ACOPF in the distribution clearing will introduce nonlinear constraints, but the derived KKT conditions cannot necessarily guarantee optimality for nonlinear programming. In this case, special techniques are used to recast the bilevel problems as a mixed-integer second-order conic programming

(MISOCP) model[44]. Given that the SOCP relaxation may be inexact, an extension of the market mechanism is proposed based on the convex-concave procedure[131].

The coupling of multi markets can introduce nonlinear terms in the objective function, such as one market's clearing quantities multiplying another market's clearing prices. The binary expansion approach[132] is introduced, where one variable is discretized and the nonlinear terms can be converted to an integer multiplying a continuous variable. This method has been widely adopted in solving joint gaming problems[94, 133].

6.1.2   Equilibrium verification and choice in EPEC

The challenges in EPEC problems originate from the choice of the equilibrium points. The EPEC solution is the Nash equilibrium of several MPEC problems, and the diagonalization or KKT reformulation is used to derive it.

However, since the MPEC problems are nonconvex, the algorithm may not necessarily converge to a locally optimal point but to a hurdle point instead. In this case, an equilibrium checking method is proposed based on diagonal optimal strategy verification [134, 135]. The MPEC problem of each firm is solved sequentially by holding rival firms' offers as the EPEC solution. If the MPEC solution is identical to the EPEC solution, it verifies the Nash equilibrium since no one wants to deviate from the point.

In addition, there could exist several Nash equilibriums for the EPEC problems, and a refined method needs to be proposed to select the one with proper economic meanings. Common choice principles include social welfare maximization and aggregated generation profit maximization[14].

6.2   Multi-agent-based models and solving methods

With the increase in multiple heterogeneous market participants, such as energy storage systems and VPPs, traditional optimization-based models have difficulty simulating such complex market equilibrium results. Multi-agent-based models and corresponding solving methods provide a numerical approach to estimating the market equilibrium. Due to the construction and expanding simplicity, an increasing number of studies focus on multi-agent-based models.

In recent years, a new trend is the emergence and enhancement of agent learning abilities. An agent with learning ability is no longer fixed to execute preset strategies but updates its strategies during the interaction in the market trading process. With the rapid development of reinforcement learning (RL) algorithms in recent years, modelling market agents' bidding behavior with RL models has attracted increasing attention. Numerous RL-based models have been used to model market participants as a part of the multi-agent simulation framework.

The main advantages of models with learning ability are as follows. First, learning ability could be used to search for the optimized bidding strategy in a complex market environment. It is simpler to build, while traditional optimization-based models rely on modelling all the market settings and solving the analytical model. Second, in complex market environments, RL-based learning agents usually need less computation time, so learning ability can be used to increase the simulation scale, even to simulate the actual market. Third, agents with learning ability are closer to the actual participant because participants in actual markets are also improving their strategies in trading interaction with the market.

Considering the need for learning, a training process is usually required in addition to the simulation process. The simulation process is conducted after the agent's model is well trained. In the

training process, the agent starts from an initial strategy and converges to an optimal strategy through a training process. The dynamic intermediate results do not represent the simulated market result. It only represents the iterative convergence process of the agent's strategy. After convergence, the final equilibrium or the following simulation equilibrium is treated as the simulated market result. This is similar to the traditional fixed strategy models because the agent's strategy is fixed in the simulation process. The main difference is that the learning model relies less on preset model parameters. In contrast, some of the model parameters are learned during the training process to improve its strategy.

Most of the current studies focus on the steady market equilibrium result. Its learning usually happens only in the training process. Some studies also allow learning in the simulation process to simulate the evolution of the agent's strategy through dynamic behavior. The significant feature of these models is that the agent's strategy may automatically change during the simulation process. This characteristic is closer to the actual participant in theory but is also difficult to control.

The reviewed literature is listed in Table 3. RL-based models account for most of the research. The Q-learning algorithm is a classic and interpretable RL algorithm that is widely used in several models to simulate the behavior of various market participants[137, 140,141,146,152,153]. However, the Q-learning algorithm is weak in dealing with the continuous bidding action space. Some studies are also investigating policy-based models, such as the deep policy gradient (DPG)[148], deep deterministic policy gradient (DDPG)[142–144, 147], multi-actor-attention-critic (MAAC)[136], and asynchronous advantage actor-critic (A3C)[138].

Although modern RL models have been widely investigated, classic RL models (such as Q-learning and Roth-Erev[145, 149, 151]) are utilized in most literature, which focus on strategy changes during the simulation process. A possible reason is that deep learning methods are still not stable and interpretable enough to simulate the strategy evolution process of market participants. There are also some models which are not based on RL framework. Some studies reach the equilibrium among agents through their own iteration rules[139, 150] or other supervised machine learning algorithm, such as support vector machines SVM[154].

7 Key issues in the gaming and equilibrium analysis

Based on the reviewed literature, this section attempts to summarize the key issues in gaming and equilibrium analysis. According to the nature of the interaction, different gaming modes should be distinguished so that the corresponding analysis instruments can be used. In equilibrium modelling, the comprehensive market mechanism should be incorporated, and the heterogeneous participants' characteristics should be considered. In addition, the model completeness and tractability constitute a trade-off, and processing techniques should strike a balance between them. How to use the growing market data to empower the equilibrium model with more predictability also remains a problem. In the dynamic equilibrium convergence, we should consider how to better capture participants' learning abilities.

7.1 The analysis of various gaming modes in different market settings

With the decarbonization and decentralization of the power system, there have been fundamental changes in the power flow, generation mix and participant structure, rendering more diversified competition and interactions.

It is essential to capture the nature of gaming and interaction in a new market setting and choose the respective instruments to analyze and compute the equilibrium.

For example, the wholesale market is organized as a centralized first-price sealed-bid auction and can be analyzed by Nash equi-

Table 3 Categorization of reviewed literature according to learning ability and technical implementation

Reviewed literature	Multi-agent	Learning ability	Learning in simulating	Learning algorithm	RL
[136]	√	√		MAAC	√
[137]	√	√		Q-learning	√
[138]		√		PPO(A3C)	√
[139]	√				
[140]	√		√	Q-learning	√
[141]		√		Q-learning	
[142]	√	√		DDPG	√
[143]	√	√		DDPG	√
[144]		√		DDPG	√
[145]	√	√	√		√
[146]	√	√	√	Q-learning	√
[147]	√	√		DDPG	√
[148]	√	√		DPG	√
[149]	√	√	√	Roth-Erev	√
[150]	√				
[151]	√	√	√	Roth-Erev	√
[152]	√	√		Q-learning	√
[153]	√	√		Q-learning	√
[154]	√	√		SVM	

librium theory. In the distribution market, the P2P equilibrium can be analyzed by a distributed algorithm. According to different DER aggregation modes, the core stability theory can be used to analyze cooperative games, and the best-response derivation can be used to analyze Stackelberg games.

Additionally, in some new market settings, the interaction mechanism has to be modelled on a case-by-case basis, and several other gaming modes, such as generalized Nash game[155] and repetitive game[156], have been applied.

7.2 A comprehensive equilibrium analysis framework incorporating multiple markets

In the equilibrium model, the market mechanism usually serves as an important setting or even a gamer. The introduction of new trading products and the growing importance of risk hedging have gradually extended the competition to multiple stages and products. To approximate real market equilibria, it is of great importance to capture the comprehensive market rules and their coupled relationships.

On the one hand, the diversified clearing rule for different markets should be modelled, including the determination of product requirements, bidding structures, clearing models and settlement rules.

On the other hand, the coupled relationship among markets should be modelled. In some cases, the market organizers directly conduct joint clearing, such as energy and reserve in PJM, and this affects the underlying market mechanism. In some cases, the markets are not cleared jointly, but the feasible bidding regions of participants are mutually coupled, which constitutes a trade-off in their decision processes.

7.3 The modelling of heterogeneous market participants' characteristics

The modelling of participants lies in the core of the equilibrium analysis. Their operating and decision characteristics have a great impact on their bidding behaviors and the formulated equilibrium.

The operating characteristics can be diversified due to the emergence of new kinds of participants, such as renewables, storage, VPP and aggregators. When analyzing the equilibrium, customized bidding problems should be formulated based on the modelling of physical operating constraints and cost functions.

Furthermore, the subjective decision characteristics could be different within a certain kind of participant. As key parameters in the bidding problem, the intro-group differences in the willingness to pay, risk attitude and rationality are nonnegligible and must be modelled carefully.

7.4 The trade-off between completeness and tractability in equilibrium

To better capture the market interactions and approximate the real equilibrium, practical factors, including multi-market clearing, uncertainty, and heterogenous participants' preferences, need to be incorporated. However, the overcomplicated model might be intractable, which calls for proper simplifications that strike a balance between practicability and tractability.

There are several techniques in the modelling step. For example, the stochastic scenarios of some variables with different values and probabilities are usually used to replace the probability distributions of the variables with imperfect information, where the latter is difficult to calculate in an optimization model.

The simplifications can also be conducted in the solution by using methods such as discretization. The binary expansion approach discretizes the power output and consumption, which narrows the feasible region and may discard the optimal value. However, the nonlinear terms are converted into mixed-integer terms, and the optimality gap will be acceptable when the discretization level is refined.

7.5 The use of data-driven methods to increase practicability

Some key parameters in the equilibrium are private information and unknown to the analyzers, and the arbitrary manual setting may cause deviation in the predicted results. Thanks to the growing availability of historic data, we can embed data-driven parameter extraction to empower the equilibrium model.

On the one hand, the physical characteristics can be learned by data mining methods such as nonintrusive detection. The operating constraints and cost functions can be modelled, and the ability to provide regulation, energy and ramping can thus be evaluated. The detection can be made more complicated by considering intertemporal coupling and cluster interaction.

On the other hand, the individual decision characteristics can be revealed, including risk preferences and rationality degrees. After modelling the decision optimization problem, reverse optimization or inverse reinforcement learning can be used to infer decision parameters from the observed decisions.

7.6 The representation of participants' learning characteristics in the equilibrium computation

With the increasingly complex and dynamic market environment, participants' bidding behavior will no longer remain the same. It will change according to the market rules, market boundaries, and other competitors' conditions. The real behaviors of market participants are a consistent learning process instead of static market equilibriums. However, current equilibrium models can barely simulate this characteristic. How to represent participants' ability to learn from the market is still an unsolved problem.

Several existing studies attempt to simulate the learning process with the agent's training process in RL. However, such simulation results have not been proven to be representative of reality. Neither is the convergence of training RL agents in a changing environment or with some changing competitors well documented or analyzed.

8   Conclusions

In the changing electricity market with decarbonization and decentralization, this article reviews the recent developments in the equilibrium analysis instruments. We review the research from the perspective of modelling methods, practical settings and solution techniques.

It is found that the modelling methods vary across different market settings, including incooperative Nash games in wholesale markets, P2P trading in distribution markets and cooperative games or Stackelberg games in DER aggregation. Compared to traditional gaming and equilibrium analysis, the novel studies pay more attention to emerging entrants, novel trading categories and multi-market participation. Additionally, the assumption of perfect information and complete rationality is relaxed, and data-driven extraction of characteristics is widely studied. We also recommend study focus on refined analytical techniques and agent-based models, which can properly address the increasing model complexity.

By gathering up the thread of relevant research, six key issues in

gaming and equilibrium analysis are summarized. First, we need to distinguish between various gaming natures and choose the proper modelling instruments. In the modelling process, we need to consider the comprehensiveness of market mechanisms and heterogeneity of participants' characteristics. The trade-off should be carefully considered, and balance needs to be struck between modelling completeness and tractability. The utilization of historical data is also a hot topic that can eliminate the weaknesses in arbitrary parameter settings. Finally, the representation of participants' learning ability is also significant, especially in a continuously changing market environment.

Potential future research directions include the modelling of new market participants and emerging trading products, the equilibrium analysis in the distribution market, the consideration of imperfect information and rationality, more practicable and tractable solution algorithms.

We hope our review provides useful references and insights into future equilibrium analysis research.

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Additional information

Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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