Analytical Approach for Simultaneous Optimal Sizing and Placement of Multiple Distributed Generators in Primary Distribution Networks

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***Abstract*—This paper presents a novel analytical expression for finding optimum sizes of multiple Distributed Generators (DGs) simultaneously in order to minimize the power loss reduction ratio. The optimal power factor based on active and reactive power demands by the load is considered for sizing multiple DGs. The generalized analytical expressions for finding optimum sizes of two DGs simultaneously are presented considering mutual coupling factor to reduce the computation time. A brief description of both Exhaustive Load Flow (ELF) method and Improved Analytical (IA) methods are also presented. Systems with varying size and complexity were used for testing and validating the proposed analytical expressions for simultaneous optimal sizing and placement of DGs for active power loss reduction. The test results of IEEE 14 and 30 bus systems using the proposed methodology were compared with ELF and IA methods. Results proved the effectiveness of the proposed method in comparison to the ELF and IA methods.**

Keywords*—loss reduction; simultaneous optimal sizing; multiple DGs; optimal power factor; coupling effect; interdependencies; Improved Analytic (IA); Exhaustive Load Flow (ELF)

I. Introduction

Due to environmental friendly nature and other economic concerns Distributed Generations (DGs) incorporation in advanced power network, has got the attention of many research communities for last decades. The intervention of DGs in today's distribution networks is because of the technological advancements in generators, power electronics devices and storage devices. DGs contribute in the application of competitive energy policies, diversification of energy resources, reduction of on-peak operating cost, deferral of network upgrades, lower losses and lower transmission and distribution costs, and potential increase of service quality to the end-customer [1].

To take more and more benefits from these potential opportunities, the placement and sizes of the DG's are very crucial. As the matter of fact, it may positively influence the existing network in loss minimization, voltage and loadability enhancement, reliability improvement and network upgrade deferral [2] [3]. Similarly, DGs have been integrated in electricity markets for ancillary services such as spinning reserve, reactive power support, loss compensation, frequency control and other fast response services. On the other hand, poorly planned and improperly operated DG units can lead to reverse power flows, excessive power losses and causing overheating of feeders [4].

DG sizing and placement for loss reduction has been an interesting research area of large number of researchers in recent past. Most of this work has been done for placing and sizing of single DG in the system, especially for the peak load hours which will not be applicable for other loading conditions. Genetic Algorithm (GA) based techniques [5, 6], Particle Swarm Optimization (PSO) [7], evolutionary programming (EP) [8] and analytical expressions methods [9-10] are few examples of such researches.

Due to increased interest in renewables and high motivation for the investors and systems operators, it is supposed that the number of DGs in a system will increase in the coming years. This fact has motivated many researchers to develop methods for optimal sizing and placement of multiple DGs. Rising numbers of DGs increases the complexity of the problem and necessitate a sophisticated approach for sizing and placement of multiple DGs to enhance the system in a best possible way. There have been very few researches considering the sizing and placement of multiple DGs for loss reduction [1]. Few recent examples of researches toward optimal sizing and siting of multiple DGs are using Kalman filter Algorithm [11 – 13], mix of discrete PSO and GA [14], EP [15], Chance Constrained Programming [16] and PSO [17]. While considering the analytical approaches for calculating the sizes of multiple DGs, the interdepending effects of multiple DG(s) on each other are usually ignored in calculations for optimum sizes, which can yield erroneous results [18]. Also, in iterative techniques, the sizes of oncoming DG(s) are calculated based on the already installed DG(s) which limits range for the best optimal sizes for the oncoming DGs. Very few works have considered the impact of optimal power factor for reducing the power loss but optimal power factor of individual DG at optimum location has not been considered at all [9], [19].

In this paper, a new analytical method for simultaneous optimal sizing of multiple DGs is proposed considering the interdepending factors along with optimal power factor for power loss minimization. The accuracy and effectiveness of the proposed method is compared with the exhaustive load flow (ELF) method and Improved Analytical (IA) method.

The rest of the paper is structured as follows. Section II details the derivation of analytical expressions for finding optimum size of multiple DGs simultaneously while considering the interdependencies. Explanation of optimal power factor and brief description of ELF and IA methods is also presented in Section II. Results and their comparison with state-of-the-art, along with necessary discussion, are presented in Section III. The important innovations and conclusion of whole study have been summarized in Section VI.

II. METHODOLOGY

This section elaborates the detailed description of the mathematical steps taken to formulate an expression for finding optimal DG sizes for given specific number of buses. The method explained here is for the case when two DGs are placed simultaneously, while considering the optimal power factor, which can be generalized for placement of any number of DGs in the system. To check the accuracy, effectiveness and time for finding exact solution, the Exhaustive Load Flow (ELF) Technique/method is used. The results validated by using the IEEE 14 and 30 bus systems with the system data given in [23] and [24] as an example. The 30 bus system used here is the part of American Electric Power Service Cooperation network which is released for electric utility industry as a standard test case for evaluating various analytic methods and computer programs for the solution of power system problems [24].

A. Distribution System Power Losses

Based on the active and reactive power injections at the buses, total real power loss in an "n" bus system is given as [19]:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\alpha_{ij}\left(P_iP_j + Q_iQ_j\right) + \beta_{ij}\left(Q_iP_j - P_iQ_j\right)\right] \quad (1)$$

Where,

$$\alpha_{ij} = \frac{R_{ij}}{|V_i||V_j|}\cos\left(\delta_i - \delta_j\right);\ \beta_{ij} = \frac{R_{ij}}{|V_i||V_j|}\sin\left(\delta_i - \delta_j\right)$$

P_i, P_j Active power injections at the ith and jth buses, respectively; Q_i, Q_j Reactive power injections at the ith and jth buses, respectively; $V_i\angle\delta_i, V_i\angle\delta_i$ Complex voltages at the ith and jth buses, respectively; $R_{ij} + jX_{ij}$ ijth element of impedance matrix [Z_{bus}]; n Total number of buses in the system.

B. Proposed Method

This sub-section introduces the extensive mathematical steps taken to finalize the expressions for calculating the size of DGs at two buses simultaneously, while considering the optimal power factor.

1) Analytical Expressions:

For a DG, reactive power injection is given as [9]:

$$Q_{DGi} = aP_{DGi} \quad (2)$$

Where,

$$a = (sign)\tan\left(cos^{-1}(PF_{DG})\right) \quad (3)$$

$sign$ = +1 or -1 For DG injecting reactive power or DG consuming reactive power; PF_{DG} is power factor of DG.

Active and reactive power injections at the bus where DG is installed are given, in terms of active and reactive power demands P_{Di} and Q_{Di}, as:

$$P_i = P_{DGi} - P_{Di} \quad (4)$$

$$Q_i = Q_{DGi} - Q_{Di} = aP_{DGi} - Q_{Di} \quad (5)$$

By substituting (4) and (5) in (1), the active power loss becomes:

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\alpha_{ij}\left\{(P_{DGi} - P_{Di})P_j + (aP_{DGi} - Q_{Di})Q_j\right\} + \beta_{ij}\left\{(aP_{DGi} - Q_{Di})P_j - (P_{DGi} - P_{Di})Q_j\right\}\right] \quad (6)$$

It can be proved that the minimum of active power loss of the system can be found if partial derivative of (6) with respect to injected real power from DG at ith bus computed equal to zero. Hence, (6) can be written as:

$$\frac{\partial P_L}{\partial P_{DGi}} = 2\sum_{j=1}^{n}\left[\alpha_{ij}\left(P_j + aQ_j\right) + \beta_{ij}\left(aP_j - Q_j\right)\right] = 0 \quad (7)$$

As mentioned earlier, the method for calculating the optimum size of two DGs is presented here. Supposing the DGs are to be placed at bus number "x" and "y", (7) can be re-written as:

$$\frac{\partial P_L}{\partial P_{DGx}} = \alpha_{xx}(P_x + aQ_x) + \beta_{xx}(aP_x - Q_x) + \alpha_{xy}\left(P_y + aQ_y\right) + \beta_{xy}\left(aP_y - Q_y\right) + \sum_{\substack{j=1\\ j\neq x,y}}^{n}\left(\alpha_{xj}P_j - \beta_{xj}Q_j\right) + a\sum_{\substack{j=1\\ j\neq x,y}}^{n}\left(\alpha_{xj}Q_j + \beta_{xj}P_j\right) = 0 \quad (8)$$

$$\frac{\partial P_L}{\partial P_{DGy}} = \alpha_{yy}\left(P_y + aQ_y\right) + \beta_{yy}\left(aP_y - Q_y\right) + \alpha_{yx}(P_x + aQ_x) + \beta_{yx}(aP_x - Q_x) + \sum_{\substack{j=1\\ j\neq y,x}}^{n}\left(\alpha_{yj}P_j - \beta_{yj}Q_j\right) + a\sum_{\substack{j=1\\ j\neq y,x}}^{n}\left(\alpha_{yj}Q_j + \beta_{yj}P_j\right) = 0 \quad (9)$$

The terms taken out of the summation are actually the coupling effect factors of both DGs on each other. In previous analytical techniques [2, 9, 18, 19, 21] this fact is not considered, which excludes this effect from calculating the best DG sizes leading to considerable errors in the results [18]. While calculating sizes of two or more DGs simultaneously, and considering the interdependencies and coupling effects, the best possible size for reducing the losses can be determined. Also, the method is not iterative, thus the simulation time is also reduced.

$$\text{Let}\begin{cases} X_x = \sum_{\substack{j=1\\ j\neq x,y}}^{n}\left(\alpha_{xj}P_j - \beta_{xj}Q_j\right) + a\sum_{\substack{j=1\\ j\neq x,y}}^{n}\left(\alpha_{xj}Q_j + \beta_{xj}P_j\right) \\ \quad = \sum_{\substack{j=1\\ j\neq x,y}}^{n}\left[\left(\alpha_{xj}P_j - \beta_{xj}Q_j\right) + a\left(\alpha_{xj}Q_j + \beta_{xj}P_j\right)\right] \\ X_y = \sum_{\substack{j=1\\ j\neq y,x}}^{n}\left(\alpha_{yj}P_j - \beta_{yj}Q_j\right) + a\sum_{\substack{j=1\\ j\neq y,x}}^{n}\left(\alpha_{yj}Q_j + \beta_{yj}P_j\right) \\ \quad = \sum_{\substack{j=1\\ j\neq y,x}}^{n}\left[\left(\alpha_{yj}P_j - \beta_{yj}Q_j\right) + a\left(\alpha_{yj}Q_j + \beta_{yj}P_j\right)\right] \end{cases} \quad (10)$$

Also, $\beta_{ii} = 0,\ \alpha_{ij} = \alpha_{ji}\ and\ \ \beta_{ij} = -\beta_{ji}$

Hence, (8) and (9) become:

$$\alpha_{xx}P_x + \alpha_{xx}aQ_x + P_y(\alpha_{xy} + \beta_{xy}a) + Q_y(\alpha_{xy}a - \beta_{xy}) + X_x = 0 \tag{11}$$

$$\alpha_{yy}P_y + \alpha_{yy}aQ_y + P_x(\alpha_{yx} + \beta_{yx}a) + Q_x(\alpha_{yx}a - \beta_{yx}) + X_y = 0 \tag{12}$$

By substituting (4) and (5) in (11) and (12) and arranging for DG sizes,

$$P_{DGx} = \frac{P_{Dx}(\alpha_{xx}\alpha_{yy} - \alpha_{yx}^2 - \alpha_{xy}\beta_{yx}a) + Q_{Dx}(\alpha_{xx}\alpha_{yy}a - \alpha_{yx}^2 a + \alpha_{xy}\beta_{yx}) + \alpha_{yy}\beta_{xy}(P_{Dy}a - Q_{Dy}) - \alpha_{yy}X_x + \alpha_{xy}X_y}{(\alpha_{xx}\alpha_{yy} - \alpha_{xy}^2)(1+a^2)} \tag{13}$$

$$P_{DGy} = \frac{P_{Dy}(\alpha_{xx}\alpha_{yy} - \alpha_{yx}^2 - \alpha_{yx}\beta_{xy}a) + Q_{Dy}(\alpha_{xx}\alpha_{yy}a - \alpha_{yx}^2 a + \alpha_{yx}\beta_{xy}) + \alpha_{xx}\beta_{yx}(P_{Dx}a - Q_{Dx}) - \alpha_{xx}X_y + \alpha_{yx}X_x}{(\alpha_{xx}\alpha_{yy} - \alpha_{xy}^2)(1+a^2)} \tag{14}$$

Let $\begin{cases} A_{xy} = a_{xx}a_{yy} - \alpha_{yx}^2 \\ aA_{xy} = a(a_{xx}a_{yy} - \alpha_{yx}^2) \end{cases}$ and $\begin{cases} C_y = P_{Dy}a - Q_{Dy} \\ C_x = P_{Dx}a - Q_{Dx} \end{cases}$

So the general form can now be written as:

$$P_{DGx} = \frac{P_{Dx}(A_{xy} - \alpha_{xy}\beta_{yx}a) + Q_{Dx}(aA_{xy} + \alpha_{xy}\beta_{yx}) + \alpha_{yy}\beta_{xy}(C_y) - \alpha_{yy}X_x + \alpha_{xy}X_y}{A_{xy}(1+a^2)} \tag{15}$$

$$P_{DGy} = \frac{P_{Dy}(A_{xy} - \alpha_{xy}\beta_{xy}a) + Q_{Dy}(aA_{xy} + \alpha_{yx}\beta_{xy}) + \alpha_{xx}\beta_{yx}(C_x) - \alpha_{xx}X_y + \alpha_{yx}X_x}{A_{xy}(1+a^2)} \tag{16}$$

Each of these equations gives the optimum size of DGs at the respective buses while inhibiting the impact of DG at the other bus location. These are generalized equations for placing two DGs simultaneously. Suppose, two DGs need to be placed at buses "5" and "6", the solution would be:

$$P_{DG5} = \frac{P_{D5}(A_{56} - \alpha_{56}\beta_{65}a) + Q_{D5}(aA_{56} + \alpha_{56}\beta_{65}) + \alpha_{66}\beta_{56}(C_6) - \alpha_{66}X_5 + \alpha_{56}X_6}{A(1+a^2)} \tag{17}$$

$$P_{DG6} = \frac{P_{D6}(A_{56} - \alpha_{56}\beta_{56}a) + Q_{D6}(aA_{56} + \alpha_{65}\beta_{56}) + \alpha_{55}\beta_{65}(C_5) - \alpha_{55}X_6 + \alpha_{65}X_5}{A(1+a^2)} \tag{18}$$

Using similar mathematical steps, the analytic expressions for optimal sizes for any number of DGs can be found.

2) Optimal Power Factor:

To make these expressions useable for practical cases, the power factor demanded from a DG is also incorporated in these expressions. A large distribution system can be summed up as a simple two bus distribution system containing a DG, as shown in Fig. 1:

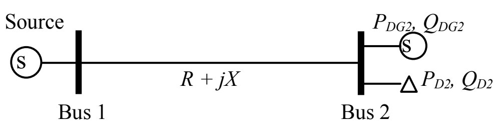

Fig. 1. Simplified two bus system [9]

Where load power factor can be given as:

$$PF_{D2} = \frac{P_{D2}}{\sqrt{P_{D2}^2 + Q_{D2}^2}} \tag{19}$$

This can be proved equal to power factor of DG, and given by:

$$PF_{D2} = PF_{DG2} = \frac{P_{DG2}}{\sqrt{P_{DG2}^2 + Q_{DG2}^2}} = \frac{P_{D2}}{\sqrt{P_{D2}^2 + Q_{D2}^2}} \tag{20}$$

Where, P_{D2} and Q_{D2} are the summed up active and reactive power demands of whole system. It is assumed that possible minimum in total losses occur at $PF_{DG2} = PF_{D2}$

Using (2) and (20), "a" can be calculated and hence the DG sizes can be computed.

3) Algorithm for Optimum Size Calculation

Fig. 2 illustrates the flow chart of the computational steps needed to find the results based on proposed analytical expressions. For the sake of illustration, case of placing two DGs is considered here.

1. Enter the number of DGs and number of buses for which optimum size needs to be calculated.
2. Run base case load flow using the (1).
3. Calculate Power factor of DG using (20).
4. Based on input data in step 1 find optimal size of DGs using expression given in (15) and (16).
5. Place DGs with size given in the system and calculated losses using (1).

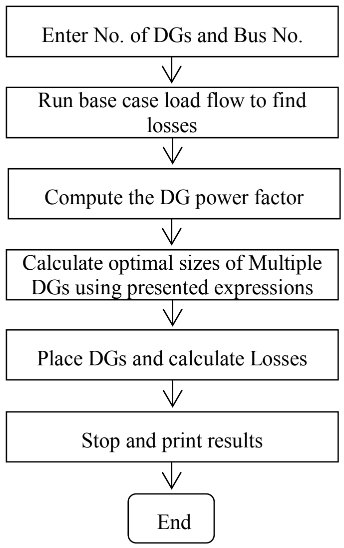

Fig. 2. Flow chart for optimum size calculations steps

C. Comparitive Studies

1) Exhaustive Load Flow

ELF method [2] is a computationally demanding but accepted technique to find optimum generator sizes for which the losses are the most minimum in the system. Hence, for validation of results and effectiveness of proposed analytical expressions, ELF method is used. To find the optimum DG sizes for respective buses (only two in each of the cases presented in this paper) at peak load demand, ELF method follows iterative method for DG placement. From 0% to 100% of load demand in steps of 0.25% of total required generation (which is actually total load demand plus losses), it places one DG at a time onto the mentioned bus and calculates the losses. In the end, the size with least losses is finalized as the optimum size.

2) Improved Analytical Method

Improved analytical (IA) method, proposed in [19], calculates the optimum sizes of multiple DGs iteratively. This relatively advanced technique is presented in the latest literature and is used here to comparison and validation of the results. The iterations for finding the optimum locations for installations of multiple DGs have been removed and only the optimum DG sizes are calculated for respective buses to reduce the computation time.

III. Results And Comparison

A. Experiments/ Usecases

Two different systems, IEEE 14 and 30 buses, with different demands and complexities have been considered for testing the proposed methodology. Details of total active and reactive power demands, and base case losses are summarized in Table I.

Fig. 3 and Fig. 4 show the comparison between the size of DGs and real power losses for 14 and 30 bus systems respectively. It is worth mentioning here that ELF method based iterative procedure was followed to place two DGs. For example, in case of 30 bus system, after placing and maintaining the best sized first DG, the calculations for finding size of 2nd DG were made.

An important conclusion which can also be drawn from Fig. 2 is that if the optimum size of DG is not placed at the respective bus, the overall losses in the system can rise resulting in inefficiency of the system, increased stress on transmission and distribution system and voltage profile issues.

Our software is developed in MATLAB command line environment which computes the base case losses, optimum sizes of multiple DGs at the bus locations given as input based on the presented methodology and losses after placing the DGs with the calculated sizes. To generalize, the DGs with ability of injection both active and reactive power with power factor of 0.96 at peak demand are considered in this software but the sizes for DGs of any type can be calculated. Also, the upper and lower bus voltage limits are kept between ±6% of nominal i.e. 1.06 p.u. and 0.94 p.u. [22]. The size of the DG can vary between a couple of kilowatts to 300 MW [25] but depends upon the capacity of the power system and can be considered maximum up to the total power demand plus the losses [2].

B. Test Results

1) 14 Bus System:

Simulation results with proposed methodology and other mentioned techniques have been summarized in Table I for IEEE 14 bus systems. The losses for base case and after placement of optimum sized DGs with various techniques along with computation time are shown in the table.

TABLE I. Base Case System Data

	14 Bus System	*30 Bus System*
Active Power demand	259.3 MW	283.4 MW
Reactive Power demand	73.6 MVar	126.2 MVar
Base Case Loss	15.10 MW + 36.15 MVar	17.6 MW + 22.24 MVar

From all these results, proposed method proves its superiority both in terms of simulation time and loss reduction. The IA method appears to be the nearest in terms of both these variable/objectives while the ELF method along with being the slowest in finding the optimum size on one side, gives poor loss reduction. For the case of 14 bus system, loss reduction is 85.53%, 75.58% and 81.85% is reported by proposed method, ELF and IA method, respectively.

Comparing the computational time for placement of two DGs with different techniques discussed here proves the supremacy of proposed methodology. The time consumed in computation by proposed methodology is less than half as compared to the IA method and is less than the 1/20th as compared to the time consumed by ELF method. Hence, the proposed method appears to be the best in respect of both the loss reduction and computational time.

2) 30 Bus System:

Table III presents the simulation results for IEEE 30 bus system in the same fashion as were done for 14 bus system.

It can be seen that the proposed methodology could find the optimum sizes for reduction of losses in the system relatively faster. Also the found sizes depict the effectiveness of the

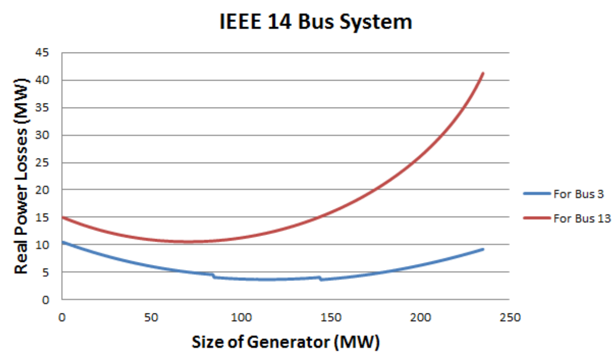

Fig. 3. Variation in losses with different DG sizes (14 Bus)

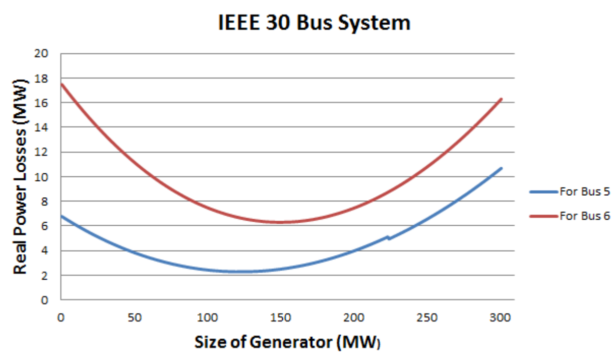

Fig. 4. Variation in losses with different DG sizes (30 Bus)

proposed analytic expression for simultaneous sizing of DGs because the loss reduction is highest among all the presented techniques. The ELF method is proved to be the least efficient in term of simulation time and optimum sizes which can reduce losses to lowest level. 87.05%, 84.84% and 85.53% are the loss reduction with proposed method, ELF method and IA method, respectively.

Proposed methodology can find the optimum size for two DGs in 30 bus system in approximately half time as compared to IA method. ELF method, being an iterative technique for finding the optimum size, consumes 25 times more time as compared to the proposed method. This extremely high simulation time can be reduced by increasing the step size from 0.25% to the 0.50% at cost of reducing the accuracy in finding optimum size.

TABLE II. DG PLACEMENT FOR IEEE 14 BUS SYSTEM WITH VARIOUS TECHNIQUES

	Proposed Method	*ELF Method*	*IA Method*
DG at bus 3 (MW)	134.2	144.8	123.8
DG at bus 13 (MW)	44.79	70.66	37.48
Losses after DGs in place (MW)	2.486	3.686	2.74
Loss Reduction (%)	83.53	75.58	81.85
Simulation Time (s)	0.42	9.39	0.90

TABLE III. DG PLACEMENT FOR IEEE 30 BUS SYSTEM WITH VARIOUS TECHNIQUES

	Proposed Method	*ELF Method*	*IA Method*
DG at bus 5 (MW)	77.413	122.657	87.775
DG at bus 6 (MW)	168.515	149.747	164.354
Losses after DGs in place (MW)	2.280	2.667	2.546
Loss Reduction (%)	87.05	84.84	85.53
Simulation Time (s)	0.52	12.98	1.01

IV. CONCLUSION

This paper has explained the placement of optimum sized multiple DGs for reducing the active power loss, simultaneously for a large scale distribution system. The generalized steps toward deriving the analytic expressions for simultaneous optimum sizing, which also include the impact of size of one DG on the other DGs those, have either already been placed or needs to be placed afterwards, are presented. The optimum power factor required from a DG is also incorporated, resulting in the versatility of the derived expressions to be used for DGs with both active and reactive power generation capabilities. Moreover, the ELF and IA methods have been presented. Although IA method was simplified for reducing the iterations and hence the simulation time, yet the proposed methodology for simultaneous optimal sizing of multiple DGs was affirmed effective and efficient in comparison with ELF and IA both in terms of minimizing the losses and simulation time. The proposed methodology is the general set of steps which can be applied for any number of DGs, but the case with only two DGs has been presented here. The location (bus number) can be input by the user. Another important conclusion is the proof for need of optimum sizing for loss reduction based on ELF method. The least losses can only be possible with optimum sized DG, otherwise the losses may go beyond the value of losses without DG(s), hence the desired benefits from DG placement cannot be achieved.

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